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ON DIQUARK EFFECTS IN HIGH - ENERGY PHYSICS

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Abstract

This thesis reports on phenomenological investigations of the hypothesis that there exist a bound state, a *diquark*, in the quark-quark interaction. Such a state is not a colour singlet, and thus, like quarks, cannot be directly observed. It could in principle appear

1) in the hadronisation process,
2) as a constituent in hadrons, notably baryons,
3) in the QCD plasma.

All these conceivable arenas for diquarks in high-energy physics are studied.

The hypothesis is found to be consistent with experimental data from high-energy lepton-lepton, lepton-hadron and hadron-hadron collisions. In some cases, the existence of diquarks seems to be the most plausible explanation available for observed data trends.

In the case of the QCD plasma, no experimental data are as of yet available, but an interesting phenomenon, Bose-Einstein condensation, is suggested to occur as a consequence of the inclusion of massive bosons, diquarks, in equilibrium in the plasma.

Descriptors : Diquarks, baryon production, deep inelastic scattering, e^+e^- annihilation, large - p_T scattering, quark matter, QCD plasma, quark-gluon plasma, Bose-Einstein condensation.

Preface

This thesis consists of two parts.

The first part contains a very brief non-technical introduction, as well as a somewhat more technical section commenting on the individual papers.

The scientific contributions are presented, in the condensed format appropriate for rapid international publication, in papers I - VI . These papers constitute the second, and major, part of the thesis.

The papers are :

I *Baryons from diquarks in e^+e^- annihilation*
Physical Review **D28** (1983) 257
(with S. Fredriksson, T.I. Larsson and M. Jändel),

II *Indications of hard diquarks in e^+e^- annihilation*
Physical Review **D30** (1984) 2310
(with S. Fredriksson, T.I. Larsson and M. Jändel),

III *Large - p_T protons from constituent diquark scattering*
Physics Letters **149B** (1984) 509
(with S. Fredriksson),

IV *New ideas on the proton - neutron differences in deep inelastic structure functions*
Physics Letters **162B** (1985) 373
(with S. Fredriksson),

V *Hadron p_T correlations in quark jets*
Physical Review Letters **56** (1986) 2428
(with S. Fredriksson),

VI *Role of diquarks in the quantum - chromodynamical plasma*
Stockholm preprint TRITA-TFY- 86 -16 (1986)
to be submitted, in condensed form, to Physical Review Letters.

Contents

PART 1

1 Introduction

The present thesis contains contributions to the field of elementary particle physics, in particular certain aspects of strong interaction phenomenology.

The strong interaction is one of the four effective types of interaction responsible for phenomena we can observe. It is, in fact, the interaction responsible for the fundamental structure of the bulk of observable matter in the universe.

The last few decades have seen tremendous theoretical and experimental progress in this field. We now think we know the building blocks of matter, namely quarks, and the theory to describe their interaction, namely quantum chromodynamics, in short QCD.

There is also a widespread sense of optimism in the scientific community as to the possibility of a unified description of the effective types of interaction, and matter, in the not-too-distant future. This is, of course, a fascinating prospect, and an enormous amount of effort is being directed towards this goal by a large number of theoretical physicists.

In view of this, it is a remarkable fact that, even though since two decades we strongly believe we know that the proton is a bound state of three quarks, and since one decade also the Lagrangian of the theory for strong interaction, we still cannot from basic theory compute the proton properties, or even prove that the proton exists!

In general, bound states in QCD, which are all we can observe, are at present not calculable from first principles. It is obvious that a phenomenological approach is also called for, in order to allow the accumulated outcome of experimental physicists' efforts to increase our understanding of nature. Phenomenological physicists strive to act as an interfacing link between theoretical and experimental endeavour.

The task of understanding the fundamental structure of nature is so difficult and so important, that every approach is needed; theoretical, experimental, and phenomenological alike.

2 Comments to the papers

A common feature of all the papers in this thesis is the notion of a *diquark.*

This is a bound state of two quarks. Of course, the existence or non-existence of such a state cannot at present be rigorously proved from first principles. One can only present arguments, more or less persuasive, for or against the hypothesis that they exist.

Several arguments have been published in favour of diquarks. See, for instance, the QCD instanton argument in **[Betman 1985]** and the string argument and the potential argument in **[Martin 1986]**.

The idea of diquarks is almost as old as the quark model **[Ida 1966]**. For a discussion with references to older work on diquarks see **[Abbott 1979]**.

Our approach has been purely phenomenological : some trends in experimental data would naturally be explained assuming diquarks to exist. We do that, and work out the predictions for other processes, and let experiment be the judge.

In order to avoid an excessive number of arbitrary parameters, we have tried to keep the model as economical as possible. An early analysis of deep-inelastic charged lepton - nucleon scattering structure functions showed that only spin-0 diquarks were needed to explain the data **[Fredriksson 1982, Fredriksson 1983]**. In view also of the fact that the QCD spin forces (in the one-gluon-exchange approximation) favour antiparallel spins, we assume that only scalar diquarks are dynamically bound. To further reduce the number of parameters, and for simplicity, we assume only the ground state to be of importance, as long as there is no phenomenological need for excited diquarks.

The (ud) diquark is naturally expected to be the lightest and to be the one most relevant to hadron physics. However, in some processes heavier diquarks ((us), (ds), (uc), (dc), (sc), ...) could also be of relevance.

Since the diquark is not pointlike, its interaction amplitude is suppressed by a form factor at each diquark vertex. We take this to be of the dipole type $F(Q^2) = M^2/(Q^2 + M^2)$ .The parameter M^2, which is related to the "pointlikeness" of the diquark, is essentially the only parameter of the model. As is seen in the papers, data seem to favour a value $M^2 \approx 10\ \mathrm{GeV}^2$, which implies that the diquark is quite small.

For a recent review with a complete bibliography of the model, see [**Fredriksson 1986**].

Diquarks are not colour singlets, and thus subject to confinement. They could in principle appear

1) in the hadronisation process
2) as constituents in hadrons
3) in the QCD plasma.

The first of these arenas for diquarks in high-energy physics is looked into in papers I, II and V, the second in papers III and IV, and the third, finally, in paper VI.

2.1 Paper I : Baryons from diquarks in e^+e^- annihilation

In this paper we analyse the role of indirect diquarks in e^+e^- annihilation.

Hadron production in e^+e^- annihilation, as in any strong colour field, is conventionally interpreted in the following way.

The electron and positron annihilate into a virtual electroweak gauge boson, at low energies almost always a photon. This state decays into a quark-antiquark pair, which separates back-to-back due to energy and momentum conservation. As a consequence of the non-Abelian structure of QCD, the gluonic colour field joining the **3** and **3*** is self-attractive, and becomes quasi - one-dimensional -- a "chromo-electric flux tube". Since the flux is constant, the energy carried by the field increases linearly with the separation distance. This provides an intuitive understanding of confinement -- the energy needed to "ionise" a colour charge would be infinite.

Instead, the field is locally screened by the pair production of a colour **3** and **3***, supposedly a new quark - antiquark pair, so that we now have two flux tubes, each less energetic than the original one. The process continues recursively until the available energy is used up, and we are left with a number of mesons in the final state.

For quantitative calculations of the pair-production probability per unit time and volume, it is customary to apply the Schwinger formula for pair creation of fermions through tunneling in a strong electric field **[Schwinger 1951, Casher 1979, Andersson 1980, Glendenning 1983]** borrowed from QED [1]:

$$W_F = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[-\frac{n\pi m^2}{eE}\right]. \tag{2.1.1}$$

However, hadron production data from e^+e^- annihilation show that baryons are produced at a rate of about 8 % compared to mesons. This is hard to accommodate in a pure quark - recombination picture. [2]

Baryon production can in a natural way be accounted for if the **3* - 3** pair produced in the field can be a diquark - antidiquark pair **[Ilgenfritz 1978, Andersson 1982]**.

In paper I we analyse baryon production taking into account the correct Schwinger formula for spin-0 bosons **[Brezin 1970]** :

$$W_B = \frac{1}{2}\frac{\alpha E^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(-1)^{n-1}\exp\left[-\frac{n\pi m^2}{eE}\right]. \tag{2.1.2}$$

It can immediately be seen that using formula (2.1.2) instead of, as had earlier been done, (2.1.1), the diquark mass needed to account for the data is decreased, due to the spin factor 1/2 and to the fact that (2.1.2) is an alternating series. The fact that in our approach only scalar diquarks are considered to be dynamical entities further enhances this effect.

Taking the field strength eE to be 0.20 GeV^2 **[Andersson 1982]**, we get $m_{(ud)}$ = 225 MeV and $m_{(us)} = m_{(ds)}$ = 450 MeV, assuming massless light quarks and the strange diquark mass excess to be entirely due to the strange quark mass.

It is interesting to note that the same diquark mass $m_{(ud)}$ = 225 MeV is what is needed to reproduce the proton mass in the MIT bag model, if the proton is considered as a bag with a massless u quark and a massive (ud) diquark.

Since the leading terms in (2.1.1) and (2.1.2) go like exp($-am^2$), where a is of the order of 10 GeV^{-2}, it is evident that charmed or heavier quarks and diquarks are fatally suppressed.

The R.H.S. of Eq. (11) in the paper is misprinted and should read 4% . This is easily understood, since each $\bar{\mathrm{D}}\mathrm{D}$ pair gives two baryons but each $\bar{\mathrm{q}}\mathrm{q}$ pair only one meson. The results, Eqs. (11) - (13), are, however, correct.

The natural prediction in paper I from our diquark model with scalar diquarks only is that decuplet baryons should be strongly suppressed in comparison with octet baryons with the same flavour content. All spin-3/2 baryons must in our picture come from multiple quark recombination or decaying spin-1/2 resonances. We thus expect such baryons to be suppressed by an order of magnitude, whereas SU(6) - symmetric diquark models would give more decuplet than octet baryons.

CLEO at CESR (Cornell) and TASSO at PETRA (DESY) have searched for the decuplet baryons Ξ^*, Δ^{++}, Σ^{*+} and Σ^{*-}. They have not seen any signal, and have set the following limits :

$\Xi^* / \Xi < 0.16$ at the 90% confidence level, both on the Υ peak and in the continuum (CLEO, [Alam 1984]),

$\Delta^{++} / p < 0.11$ (95% CL) and

$(\Sigma^{*+} + \Sigma^{*-}) / \Lambda < 0.30$ (95% CL)

(TASSO at 34 GeV, [Althoff 1984]).

2.2 Paper II : Indications of hard diquarks in e^+e^- annihilation

In this paper we analyse the role of direct diquarks in e^+e^- annihilation.

The total hadronic cross section [3], normalised by the $\mu^+\mu^-$ cross section

$$R = R(W) = \frac{\sigma(e^+e^- \longrightarrow \text{hadrons})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)} \tag{2.2.1}$$

has been a valuable (and supportive) test of the quark model, since over the threshold $W = 2m_j$ each quark flavour j contributes a term [4]

$$\Delta R_{q_j} = 3e_{q_j}^2 \left[1 - \frac{4m_{q_j}^2}{W^2}\right]^{1/2} \left[1 + \frac{2m_{q_j}^2}{W^2}\right] \tag{2.2.2}$$

to R , a very distinct signature, since the kinematic threshold factor in (2.2.2) is very close to a step function. The factor 3 is for colours. $W = 2E_{\text{beam}}$ is the total energy.

If one takes into account the possibility of direct $\bar{D}D$ production, i.e. the direct coupling $\gamma^* \to \bar{D}D$, one gets additional contributions for each scalar diquark (D_i = (ud), (us), (ds), (uc), (dc),...)

$$\Delta R_{D_i} = \frac{3}{4} e_{D_i}^2 \left[1 - \frac{4m_{D_i}^2}{W^2}\right]^{3/2} F_i^2\left(W^2\right). \tag{2.2.3}$$

We show in paper II that there is room in the data for this contribution, as diquarks are suppressed by

1) a factor 1/4 due to spin,
2) an extra kinematic phase - space factor $1 - 4m_i^2/W^2$, since they must be produced in P - wave, and
3) the diquark timelike form factor squared $F_i^2(W^2)$.

However, diquarks are shown not to be negligible altogether. We argue that the charmed diquarks (uc), (dc) and (sc) should, due to their high summed charge squared, give a contribution on the level of 10% in the region $W =$ 4-8 GeV. We suggest this to be the explanation of the broad structure in R, which has puzzled the physics community **[Barnett 1980]**, in this region. It has been found, using smearing techniques for both theoretical and experimental values **[Poggio 1976]**, that naive theory underestimates the data by around 15% in this region, whereas for other W the discrepancy is only 7%, which is interpreted as the contribution of perturbative 3-jet - events.

The dominant process from direct diquark production is (uc) pair production, which would give very clear experimental signatures in this region :

1) leading Λ from Λ_c decay,

2) a more transverse jet angular distribution, especially when triggering on Λ, since scalars are produced with a $1 - \cos^2\theta$ distribution, as opposed to $1 + \cos^2\theta$ for fermions,

3) back-to-back baryon-antibaryon correlations.

2.3 Paper III : Large - p_T protons from constituent diquark scattering

In this paper, we analyse the role of constituent diquark elastic scattering in pp collisions, with respect to large - p_T proton production.

Experimental data from the Split Field Magnet at the CERN ISR **[Breakstone 1984a]** show that the fractional proton yield in $\sqrt{s}$ = 62 GeV pp collisions is very high in some regions of phase space, and falls off rapidly with increasing transverse momentum, p_T, and cms angle, θ. (See Fig. 2 in Paper III.) The ratio of protons to all positive particles is seen to be as high as 50% for low p_T and θ, and falls to an asymptotic level of a few % as p_T and θ increase. However, the ratio of antiprotons to all negative particles is at the few % level, and does not depend significantly on the kinematic variables.

Particle production in pp collisions is customarily interpreted in terms of parton elastic scattering with subsequent hadronisation by much the same mechanism as the one outlined in section 2.1. Thus, the baryon production rate is expected to be due to the probability for production of a $\bar{D}D$ pair as compared to a $\bar{q}q$ pair, which is known from e^+e^- data to be around 4% all over phase space. This accounts for the antiproton rate, but parton models with only quarks and gluons would have severe difficulties in explaining the proton production characteristics outlined above.

However, taking into account the possibility of diquarks as partons which can scatter collectively, the observed proton behaviour is shown to be a natural consequence of diquark elastic scattering. In particular, the fall-off of the fractional proton rate with the kinematic variables is claimed to be due to the Q^2 - dependent form factor

$$F\left(Q^2\right)=\frac{1}{1+Q^2/M^2}. \tag{2.3.1}$$

The value for the diquark size parameter M^2 favoured by the data is 10 GeV^2, the same as the one obtained earlier by analysing the scaling violation in deep inelastic lepton - nucleon scattering structure functions **[Fredriksson 1982, Fredriksson 1983]**.

The experimental group came to essentially the same conclusion, $M^2 = 10\text{-}20\ \mathrm{GeV}^2$, independently in an analysis assuming constituent bound diquarks with the same form factor, but different scattering amplitudes and fragmentation functions **[Breakstone 1984b]** . They can rule out diffractive proton scattering. Extra support to the notion of constituent diquark scattering is also provided by the quantum - number correlations observed, namely that baryon number tends to disappear from the forward direction when triggering on a large - p_T proton **[Fischer 1985]** . This seems to rule out the possibility that most large - p_T protons would be produced by the same mechanism as large - p_T pions, i.e. through scattering of one single quark.

An earlier Split Field Magnet collaboration found **[Drijard 1979]** that the inclusive distribution of particles in the spectator jet, when triggering on a proton at $\theta = 20°$, could only be described by single quark, as opposed to "diquark" or gluon, fragmentation. This also would indicate that the triggering proton contains two valence quarks from the incident proton.

Earlier, our model has also been used **[Larsson 1984]** to explain large - p_T proton production data in $\pi^- p$ interactions at Fermilab **[Frisch 1983]** .

There are also other diquark model analyses **[Laperashvili 1982, Minakata 1980, Sosnowski 1983]** of various large - p_T proton data, and an upcoming analysis by Efremov and Kim at Dubna of new 70 GeV Serpukhov data **[Efremov 1986]**.

Further, new data from Fermilab seems to have interesting signatures **[Jaffe 1986]** .

2.4 Paper IV : New ideas on the proton - neutron differences in deep inelastic structure functions

In this paper, we analyse various combinations of structure functions in deep inelastic charged lepton - nucleon scattering within the frameworks of perturbative QCD and our diquark model, and propose how future experiments can discriminate between these two possible scale-breaking contributions.

It is a well-known fact that the structure function F_2 in the expression for the charged lepton - nucleon scattering cross section

$$\frac{d^2\sigma}{dQ^2 d\nu} = \sigma_{\mathrm{Mott}} \left\{ \frac{Q^2}{4ME^2} F_1 + \frac{1}{\nu} \left(1 - \frac{\nu}{E}\right) F_2 \right\}, \qquad (2.4.1)$$

which in the naive quark-parton model should scale, i.e. only be a function of $x = Q^2/(2M\nu)$, also turns out to depend on Q^2. This is the standard effect of scaling violation in deep inelastic structure functions, and is traditionally interpreted in terms of perturbative corrections to the quark distribution functions in the nucleon, i.e.

$$F_2(x) = \frac{4}{9} x\, u(x) + \frac{1}{9} x\, d(x) \longrightarrow \frac{4}{9} x\, u\left(x, Q^2\right) + \frac{1}{9} x\, d\left(x, Q^2\right) = F_2\left(x, Q^2\right), \qquad (2.4.2)$$

(apart from the "sea" of quark-antiquark pairs at low x). The numerical coefficients are the quark charges squared.

These perturbative corrections can qualitatively be understood by noting that at higher Q^2, the photon resolves the shorter-distance structure of the "dressed" quark, and the probability is higher that the quark will have radiated a gluon and lost a portion of its momentum, which would explain why F_2 decreases with Q^2 for large x. These corrections are quantified by the Altarelli - Parisi equations for the evolution of the parton distributions with Q^2 [Altarelli 1977].

Another possible cause of scaling violation is constituted by non-perturbative collective parton phenomena, or higher twists, which give contributions vanishing as powers of Q^2, as $Q^2 \rightarrow \infty$.

For example, at very low Q^2, when the virtual photon wavelength is greater than the nucleon size, the internal structure is not resolved, and the whole nucleon collectively absorbs the photon. This process is characterised by the nucleon charge, and by the nucleon form factor which falls rapidly for Q^2 above 0.7 GeV^2. Similarly, in Rutherford's classical elastic α - nucleus scattering experiments, Q^2 was high enough to probe the interior of the atom, but low enough that the nucleus was essentially pointlike, and the photon coupled to the total nucleus charge Z.

Deep inelastic nucleon scattering is interpreted in terms of elastic parton scattering, and a relevant higher-twist effect would be the collective coupling of the photon to a pair of quarks, a diquark **[Schmidt 1977, Donnachie 1980, Fredriksson 1982]**. Diquarks correspond to the twist-six term in the operator product expansion. The diquark electromagnetic form factor is taken to be of the dipole form in Eq. (2.3.1).

The observed scaling violation can be fitted with perturbative corrections only, or with diquark contributions only. A realistic model should incorporate both, but how do we determine the relative importance? In the future data from HERA, which will extend to very high Q^2, higher twist contributions will have vanished and the only remaining Q^2-dependence will be the perturbative corrections, which are logarithmic in Q^2.

However, in paper IV we suggest how to use existing or near-future data to discriminate between perturbative and non-perturbative scaling violations, by studying the structure function combinations $F_2{}^{\mathrm{p}} - F_2{}^{\mathrm{n}}$ and $F_2{}^{\mathrm{n}} / F_2{}^{\mathrm{p}}$, as well as $R = \sigma_{\mathrm{L}}/\sigma_{\mathrm{T}}$.

For instructive purposes, we work out the predictions for the two extreme cases, i.e. that all scaling violation is due to perturbative gluon effects and to non-perturbative diquark effects, respectively.

In the pure diquark model, the structure functions are

$$F_2{}^p = \frac{4}{9}\,x\,u(x) + \frac{1}{9}\,x\,D(x)\;F^2\left(Q^2\right) + \frac{5}{9}\,x\,q_D(x)\left[1 - F^2\left(Q^2\right)\right], \qquad (2.4.3)$$

$$F_2{}^n = \frac{1}{9}\,x\,u(x) + \frac{1}{9}\,x\,D(x)\;F^2\left(Q^2\right) + \frac{5}{9}\,x\,q_D(x)\left[1 - F^2\left(Q^2\right)\right]. \qquad (2.4.4)$$

$D(x)$ is the (ud) diquark distribution, and $q_D(x)$ that of any of the quarks in the diquark. In this approach, the entire Q^2 dependence is attributed to the diquark form factor.

We immediately see that

$$F_2^p - F_2^n = \frac{1}{3}\, x\, u(x)\,, \tag{2.4.5}$$

independent of Q^2, whereas the ratio F_2^n / F_2^p will be Q^2 - dependent.

In the pure perturbative approach, we expect the opposite, namely $F_2^p - F_2^n$ to be Q^2 - dependent and F_2^n / F_2^p to be Q^2 - independent! This is due to the fact that the perturbative corrections, given by the Altarelli - Parisi equations, to the quark distributions are flavour-independent, and that the resulting distributions $q(x, Q^2)$ almost factorise.

In paper IV, the predictions are worked out in both approaches, and compared with experimental data from the CERN European Muon Collaboration (EMC) and Stanford Linear Accelerator Center (SLAC), where in each x bin the average Q^2 is much higher in the data from EMC than in the SLAC data.

For the perturbative approach, we used the parametrisation of Duke and Owens **[Duke 1984]**, and for the non-perturbative approach we used our diquark model.

It is seen that it is hard to understand the observed difference in F_2^n / F_2^p between the two data sets without diquark effects (*see* Fig. 2 in paper IV).

A further test of the importance of diquark contributions is the cross-section ratio for longitudinal to transverse photons $R = \sigma_L/\sigma_T$. This quantity can be see as a direct measure of the charged bosonic content of the nucleon, i.e. diquarks **[Abbott 1979]**, since the contribution from spin-1/2 quarks to σ_L is zero [5].

As can be seen in Fig. 3 of paper IV, R seems to be consistently "too high" at high x, although the error bars extend more or less down to zero. We show the predictions of our diquark model for various fixed Q^2. It is seen that the effect is enhanced in deuterium. This is simply due to the fact that in a neutron the relative suppression of diquark scattering due to charge is absent.

The new muon collaboration at CERN will in high-statistics measurements on deuterium and hydrogen throw more light on these matters **[Allasia 1985]**.

2.5 Paper V : Hadron p_T correlations in quark jets

This paper is a comment to an article **[Aihara 1985]** by the TPC collaboration at PEP. The experimental group has studied the angular distribution of protons produced in e^+e^- annihilation with respect to the jet axis, as well as transverse momentum correlations within proton-antiproton pairs, in order to discriminate between fragmentation models. The angular distribution analysis rules out the so-called QCD cluster model as a relevant production mechanism for baryons in e^+e^- annihilation, whereas diquark models were found to be in line with data.

Regarding transverse momentum, p_T, correlations, proton-antiproton pairs were found more often than not to come out on the same side of the jet. The experimental group interpreted this as evidence in favour of the so-called popcorn model **[Casher 1979, Andersson 1985]**. The key idea of this model is that in the chromo-electric flux tube, discussed in section 2.1, occasionally a quark-antiquark pair of the "wrong" colour is created, so that screening is not effectuated. In this case, the "new" quark follows the "old" quark instead of the "old" antiquark, and an additional quark-antiquark pair production is needed for the tube to fission into two colour-singlet tubes. This two-step mechanism was proposed by Casher *et al.* as a possible source of baryons. The Lund group subsequently extended the idea to incorporate the possibility of one or several mesons "popping up" between the baryon and antibaryon.

It was claimed in **[Aihara 1985]** that in a diquark model for baryon production one would expect an anticorrelation in p_T between the $\bar{p}$ and p ; $\alpha = -1/2$, where α is defined in paper V. However, as we argue in paper V, this is not necessarily true for baryons, even though it is for neighbouring mesons (which, incidentally, are too numerous to allow experimentally for pairing).

The p_T of hadrons in quark jets is due to internal recoil within the produced pair. An early breakup gives a p_T to the the subsequent tube, which provides a positive correlation to the hadrons originating from that tube, whereas the final breakup gives a negative contribution to the final neighbouring hadrons. The balance of these effects determines the final correlation.

A simple recursive calculation gives $\alpha = -1/2$ for mesons. However, for baryons, we expect α to be greater than $-1/2$, but smaller than $+1/2$.

The reason is that a diquark-antidiquark pair should be created with a weaker internal recoil than a quark-antiquark pair, because of spin forces. It is easy to see that a produced $\bar{q}q$ pair must, in order to have vacuum quantum numbers $J^{PC} = 0^{++}$, be produced with parallel spins, which gives a strong repulsion due to the QCD spin-spin interaction. For a produced scalar $\bar{D}D$ pair, this spin repulsion is absent.

This argument shows that mesons between baryons are not necessarily needed to explain these data, when spin-dependent forces are taken into account. There are also various other data that support the diquark mechanism for baryon production. See, e.g.,
[Bowcock 1985].

2.6 Paper VI : Role of diquarks in the quantum-chromodynamical plasma

In this paper I analyse, using methods from statistical thermodynamics, the properties of a QCD plasma containing diquarks in addition to quarks and gluons.

It is a, albeit so far only theoretically, well-established fact that at high temperatures and/or high baryon-number densities, a new state of matter should exist. (For a recent popular review, see **[Satz 1986]** .) This state, called quark matter, or, alternatively, quark-gluon plasma, or QCD plasma, is characterised by the absence of confinement on the hadronic level, as well as by the restoration of chiral symmetry, i.e. that quarks appear quasi-free over a larger volume, and practically massless instead of as "dressed" quarks with constituent masses.

It is widely believed that at temperatures above 200 MeV, corresponding to $2 \cdot 10^{12}$ K, this phase will dominate. Thus, the whole universe was a quark-gluon plasma during the first ten or so microseconds of its history **[Baym 1985]** . That was a long time ago, however, and now the temperature is nowhere near that high anywhere, except perhaps in supernovae. Instead, experimental efforts are being made to recreate this state of matter, on a much more modest space-time scale, in relativistic nucleus-nucleus collisions **[Stock 1985]** .

The plasma properties have been studied theoretically, using thermodynamical, lattice QCD, hydrodynamical, and other methods. However, so far the only coloured plasma components studied have been quarks and gluons. But if diquarks exist as dynamical objects, and if they are as pointlike as our earlier studies suggest, they should also play a role in the QCD plasma.

This idea has been proposed earlier **[Ekelin 1985]** , and in **[Ekelin 1986]** I presented a simplistic estimate of the relative abundance of diquarks in the plasma, noting that the analysis could be considerably improved.

In paper VI I present a calculation of the baryon-number rich QCD plasma properties as functions of temperature, treating the plasma as a relativistic gas of gluons, quarks, antiquarks, diquarks and antidiquarks. Interactions are to a first approximation taken into account only by introducing an energy density B and a pressure $-B$ to the perturbative vacuum, as compared to the real QCD vacuum, as in the MIT bag model. Assuming thermal and chemical equilibrium, the thermodynamic properties of the plasma components are calculated using standard stastistical methods.

It is seen that the diquark component is favoured by Bose statistics at high densities, i.e. high temperatures. This quantum-statistical effect is found to predominate over the kinematical disfavouring of diquarks due to rest-mass effects at high temperatures.

Perhaps the most interesting result is that, due to the inclusion of massive bosons in equilibrium, a Bose-Einstein condensate **[Einstein 1925]** of diquarks can form.

Such condensation is seen to occur *above* a "critical" temperature, in contrast to the well-known situation in superfluidity and superconductivity, where condensation takes place *below* a certain temperature. This is due to the evolution relations used; in the "classical" cases of constant volume, the chemical potential μ increases and reaches saturation as temperature decreases, whereas in the plasma case of constant entropy, μ is seen to increase with temperature.

It could be expected that such a "superfluid" component should have important implications for the properties of the plasma.

Footnotes

1 Schwingers calculation of the vacuum persistence probability $|\langle 0_+ | 0_- \rangle|^2$, which led to the expression for pair production, made use of the assumption that the mutual interaction of the created pair can be neglected. When applying the formalism to the present situation, this assumption is not justified. In fact, the field between the pair-produced quarks is equal in magnitude, but opposite in direction, to the external field, which is just why the flux tube fissions.

However, it has been shown **[Glendenning 1983]** that taking this into account, and assuming that the colour field is confined by an external vacuum pressure, exactly the same result is reproduced. It is interesting to note that a problem intractable in QED has been solved in QCD, because of confinement.

2 Attempts have been made, however. I will come back to the "QCD cluster" model and the "popcorn" model in section 2.5.

3 After subtraction of the contribution from $\bar{\tau}\tau$ production.

4 The muon mass has been neglected in (2.2.2) and (2.2.3).

5 Except for kinematical and constituent transverse momentum effects at low Q^2, which in practice also means low x.

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PART 2

Paper I

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Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication may be no longer than 3½ printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but, because of the rapid publication schedule, publication is not delayed for receipt of corrections unless requested by the author.

Baryons from diquarks in e^+e^- annihilation

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We find experimental support for the view that diquarks appear only as spin-0 objects. When their production rate in the color field of a quark from e^+e^- annihilation is described by the appropriate Schwinger formula for scalars, it turns out that they must be substantially lighter than earlier believed in order to explain the baryon yield.

Baryon production in e^+e^- annihilation is a sensitive probe of fundamental quark processes. About 10% of all hadrons from such reactions are baryons, and this is most likely too much to be understood in a quark-recombination picture. A recent analysis[1] indeed shows that at least 90% of the baryons must come from other sources, although another opinion has been presented within a more complicated recombination scheme.[2] The most widespread explanation of the rather large baryon yield in high-energy e^+e^- processes is, however, that baryons come from *diquarks*,[3,4] and the aim of this work is to learn about those diquarks from the scarce data.

Diquarks, i.e., tightly bound quark pairs, can in principle appear on two levels in e^+e^- annihilation: *direct* ones from $e^+e^- \to D\bar{D}$, where the diquarks D and $\bar{D}$ fragment to hadrons, or *indirect* ones from $e^+e^- \to q\bar{q}$, followed by a quark fragmentation like $q \to q(D\bar{D})$ before the hadronization stage. We will not consider the complication that diquarks might be created in a collective fashion, and then break up before fragmenting to hadrons.[5] A diquark is therefore assumed always to end up in a baryon (neglecting possible $D\bar{D}$ bound states).

Only the indirect diquarks have so far been analyzed in the literature, since it has been assumed that the direct ones are strongly suppressed by unfavorable electromagnetic form factors. In addition, the direct diquarks would mostly escape in baryons that are too fast to be identified. We still believe that direct diquarks give very interesting signatures in existing data, but since they pose a different problem than the indirect ones, we will return to them in a forthcoming and more detailed work, and concentrate here on the $D\bar{D}$ pairs created in the color field from a directly produced quark-antiquark pair.

Earlier it has been taken for granted that diquarks are SU(6) symmetric and rather heavy, so that the agreement with data is a result of a delicate balance between the number of different diquarks and their best-fit masses.

Here we would like to point out that an orthogonal, and more economical, diquark model fits the data on baryon yields equally well. We assume that only spin-0 pairs can form bound diquark systems, and that these are substantially lighter than earlier anticipated. This rather extreme view on diquarks is a result of our earlier analyses of the nucleon as a bound quark-diquark system. When investigating deep-inelastic structure functions, we found[6,7] that nucleons are nearly always in $q(ud)_0$ configurations, with the $(ud)_0$ being a bound spin-0 diquark. The small fraction of spin-1 diquarks can be explained[7] as "accidental." We argued that a spin-1 system is not bound, but that the photon nevertheless can interact with such an entity whenever the lone quark happens to be so close to one of the quarks in the "true" $(ud)_0$ diquark that the photon cannot dissolve a "false" spin-1 system. Such a picture is consistent with the best-fit values both for the admixture of spin-1 diquarks in the proton wave function and for their form factor, which is much less pointlike than that of the $(ud)_0$.

If our interpretation of data from deep-inelastic scattering is correct, so that only spin-0 diquarks exist as dynamically bound two-quark systems, there is obviously no room whatsoever for spin-1 diquarks in e^+e^- annihilation. Therefore, we expect the lightest diquarks $D_1 \equiv (ud)_0$, $D_2 \equiv (us)_0$, and $D_3 \equiv (ds)_0$ and their antidiquarks to be responsible for the bulk of identified baryons. Heavier $D\bar{D}$ pairs are suppressed in the vacuum, and appear only as directly produced diquarks at high energies. Their influence on data in general will therefore be considered in our forthcoming work.

Another result of Refs. 6 and 7 is that the $(ud)_0$ is surprisingly pointlike, with a mean radius being around one third that of the proton. The $(ud)_0$ is therefore confined to about 3% of the nucleon's volume. Since it, in addition, has a momentum distribution in the proton that is only a bit more extended towards high momenta than the distribution of the lone u quark, we suspect the $(ud)_0$ to be very light. It is, however, impossible to get a more quantitative estimate of the $(ud)_0$ mass from deep-inelastic scattering data. One can, on the other hand, derive a more model-dependent value from the MIT bag model, by assuming that the proton is a u quark and a $(ud)_0$ diquark moving freely within the MIT bag. With the normal values of other model parameters, and assuming that the color-magnetic contribution is absorbed in the diquark mass, we need a $(ud)_0$ mass of 225 MeV to reproduce the proton mass of

938 MeV. Since the production rate of $D\bar{D}$ pairs from the color field in e^+e^- annihilation is very sensitive to the D mass, the internal consistency of the model hence requires a best-fit value of 200 – 300 MeV for the $(ud)_0$ mass.

The standard way of estimating the number of fermion-antifermion pairs created in a strong color field is to apply the celebrated Schwinger formula,[8] as borrowed from quantum electrodynamics:

$$W_F = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE}\right) . \tag{1}$$

W_F is the probability of pair creation per unit time and volume, α is the fine-structure constant, eE the strength of the field, and m the mass of the fermion.

It seems to have been overlooked in the current diquark literature, however, that *Eq. (1) is not valid for spin-0 diquark pairs.* For such bosons the correct Schwinger formula[8] reads

$$W_B = \frac{1}{2} \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} (-1)^{n-1} \exp\left(-\frac{n\pi m^2}{eE}\right) . \tag{2}$$

The crucial difference between (1) and (2) is the trivial spin factor $\frac{1}{2}$ in (2), a feature that would remain in any reasonable QCD modification of (1) and (2). A questionable, but widespread, simplification is to use only the first term in (1) for the massless u and d quarks. This gives an error of about 40%, and is not in line with the interpretation in the original literature,[8] where it is pointed out that the nth term in the sum is *not* equal to the probability to produce n simultaneous $q\bar{q}$ pairs.

An obvious effect of using (2) instead of (1) for scalar diquarks is that considerably lower masses are needed to fit the baryon yields.

In order to estimate the production rates of quarks and diquarks, we assume that the pair creation is mediated by low enough momentum transfers, so that the D_1, D_2, and D_3 form factors can be safely set equal to unity. The parameters in (1) and (2) that we need to fix are therefore the field energy eE per unit length in the color flux tube and the quark and diquark masses m_u, m_d, m_s, m_{D_1}, m_{D_2}, and m_{D_3}. We assume first that

$$m_u = m_d = 0 \tag{3}$$

for simplicity, and that

$$m_{D_2} = m_{D_3} \tag{4}$$

from isospin symmetry. The field strength $F = eE$ can be related to the universal Regge slope α' through[9]

$$F = eE = \frac{1}{\pi\alpha'} . \tag{5}$$

With the standard value $\alpha' = 0.90$ GeV^{-2},[10] one gets the alternative

$$F_1 = 0.35 \text{ GeV}^2 . \tag{6}$$

After a more detailed analysis of the bulk of e^+e^- hadron data, the Lund group argued,[3] however, that the smaller value

$$F_2 = 0.20 \text{ GeV}^2 \tag{7}$$

is needed for a good fit to data. We will test both options.

The mass m_s can be related to the mean number of K^0's per event, which leads to the following ratios for the rates of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ pairs at the energy $\sqrt{s} = 34$ GeV (Ref. 11):

$$W(u\bar{u}):W(d\bar{d}):W(s\bar{s}) = 1:1:(0.3 \pm 0.1) . \tag{8}$$

Disregarding the experimental uncertainty in (8), we get from Eq. (1) that

$$m_s \approx \begin{cases} 300 \text{ MeV for } F_1 \text{ in (6)} , \\ 225 \text{ MeV for } F_2 \text{ in (7)} . \end{cases} \tag{9}$$

Next we assume for simplicity that

$$m_{D_2} = m_{D_3} = m_{D_1} + m_s , \tag{10}$$

so that the mass excess in the strange diquarks D_2 and D_3 is caused entirely by the nonzero mass of the strange s quark.

The only remaining parameter is the $(ud)_0$ mass. It can be fixed to reproduce the ratio B/M of baryon to meson yields. At PETRA energies it is known[11] that this ratio grows somewhat with increasing hadron momentum, but stays at about 8% at momenta that are low enough to ensure that the outgoing hadron does not contain a directly produced quark or diquark.

We hence have

$$\frac{W(D_1\bar{D}_1) + W(D_2\bar{D}_2) + W(D_3\bar{D}_3)}{W(u\bar{u}) + W(d\bar{d}) + W(s\bar{s})} \approx 8\% , \tag{11}$$

and (1) and (2) therefore give

$$m_{D_1} \approx \begin{cases} 300 \text{ MeV for } F_1 , \\ 225 \text{ MeV for } F_2 , \end{cases} \tag{12}$$

and, consequently,

$$m_{D_2} = m_{D_3} \approx \begin{cases} 600 \text{ MeV for } F_1 , \\ 450 \text{ MeV for } F_2 . \end{cases} \tag{13}$$

The relative frequencies of quarks and diquarks become

$$W(u\bar{u}):W(d\bar{d}):W(s\bar{s}):W(D_1\bar{D}_1):W(D_2\bar{D}_2):W(D_3\bar{D}_3)$$

$$\approx 1:1:0.3:0.14:\begin{cases} 0.02:0.02 \text{ for } F_1 , \\ 0.01:0.01 \text{ for } F_2 . \end{cases} \tag{14}$$

The existing data on the mean number of baryons in e^+e^- annihilation are therefore in line with our diquark model with maximal SU(6) breaking, i.e., no spin-1 diquarks, and with a very low $(ud)_0$ mass. In order to make more detailed comparisons with data we would also need to make much more specific assumptions, and the basic features of the model would not be as clearly probed. An example is given by the rate of Λ's and their momentum distribution. Here we would need to take into account not only all the different ways to form a Λ from indirect diquarks, but also the "leakage" from both direct diquarks and decaying heavier baryons like the Λ_c. The latter problem has been studied in Ref. 12.

The most crucial prediction for testing our model is naturally that there can be *no spin-$\frac{3}{2}$ baryons* from diquarks. All decuplet baryons must therefore come from recombination of quarks or from the creation of heavier spin-$\frac{1}{2}$ resonances that decay to spin-$\frac{3}{2}$ baryons. Both alternatives are rather improbable, and we hence expect the yields of spin-$\frac{3}{2}$

baryons to be an order of magnitude lower that those of spin-$\frac{1}{2}$ baryons. The clearest case should be the $\Sigma(1385)$, since it is comparatively simple to detect. We predict, for instance, that

$$\sigma(e^+e^- \rightarrow \Sigma(1385)) << \sigma(e^+e^- \rightarrow \Lambda(1115)) \quad ,$$

while SU(6) symmetry would lead to about three times as many $\Sigma(1385)$ as directly produced $\Lambda(1115)$.

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Paper II

Indications of hard diquarks in e^+e^- annihilation

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It is suggested that very small spin-0 diquarks are directly produced in e^+e^- annihilation and then fragment into leading baryons and other hadrons. The most influential diquark is the charmed diquark (uc), due to its high charge. It gives a sizable contribution to the hadronic R factor and to the two-jet angular distribution in the energy region $W=4-8$ GeV. At these energies, a careful study of Λ production would provide the best additional test of the model.

I. INTRODUCTION

Recently, we have suggested in several publications[1-5] that diquarks are responsible for many interesting trends in high-energy data. In this paper we continue the analysis along these lines by investigating the role of diquark-antidiquark $(D\bar{D})$ production in e^+e^- annihilation. We then refer to those $D\bar{D}$ pairs that are directly produced by the virtual photon, in contrast to the "vacuum" pairs created in the color field of a produced quark-antiquark $(q\bar{q})$ pair. The latter case has been studied by us earlier.[3]

Obviously, the direct process $e^+e^-\to D\bar{D}$, followed by the D and $\bar{D}$ fragmenting into hadrons, can be of importance only if the diquarks are pointlike enough to compete with the dominating quark process $e^+e^-\to q\bar{q}$. In most theoretical analyses of e^+e^- annihilation such $D\bar{D}$ pairs are neglected, either because they are supposed not to exist at all, or because the diquarks are considered so large that they are suppressed by very small form factors. There are, however, a few suggestions in the literature that directly produced diquarks might give measurable effects,[6-9] but no effort has been made to probe their relative importance or sizes by analyzing the data.

Such an analysis can be made in a fairly straightforward way, however. As an input for predictions we will use only the particular diquark model that we derived earlier when analyzing the data on deep-inelastic lepton-nucleon scattering.[1,2] This will result in a reproduction of the data on the hadronic R factor, $\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$, as well as predictions for the two-jet angular distribution and the baryon yields; all as functions of the total e^+e^- collision energy W. We will not make any attempt to get a better fit to data by adding perturbative QCD corrections. Instead we follow the philosophy discussed in Ref. 5. There we noted that data from, for instance, deep-inelastic lepton-nucleon scattering can be reproduced both with perturbative QCD effects only and with nonperturbative diquark effects only, and hence with any mixture of the two. Any admixture of diquark effects would therefore mean that there is a lower "need" for a gluonic correction, and hence that the strong coupling constant is lower than the values extracted from conventional perturbative QCD fits to those data. Consequently, it seems most straightforward to take the diquark concept to its limits by neglecting perturbative QCD corrections to the processes under study. By concentrating, in addition, on the features where diquarks would give particularly clear signatures that would be hard to understand as coming from gluons, one will (we hope) be able to find out whether diquarks exist or not as dynamical objects.

II. THE DIQUARK MODEL

The main assumption in our diquark model is one of simplicity, namely, that genuine diquarks, i.e., dynamically bound two-quark states, appear only as spin-0 objects, which are quite small.

Our fits[1,2] to the data on deep-inelastic lepton-nucleon scattering showed that the dominant diquark in nucleons is the $(ud)_0$ with spin and isospin 0 and SU(3) color representation $\underline{3}^*$. Spin-1 configurations of two quarks give non-negligible contributions due to the high electric charge of a uu pair. Such two-quark systems turn out, however, to be so rare, heavy, and large that we guess they are accidental in nucleons, i.e., they do not appear as bound objects. Instead they couple to the incoming photon only "by accident" when the single quark happens to be so close to one of the quarks in the genuine diquark that the photon cannot dissolve the unbound two-quark system. Hence, spin-1 diquarks do not appear in e^+e^- reactions.

The $(ud)_0$ seems to have an electromagnetic form factor of the type

$$F_{(ud)_0}(Q^2)=(1+Q^2/M^2)^{-1} \quad \text{for spacelike } Q^2>0\,, \qquad (1)$$

with $M^2\approx 10$ GeV^2, corresponding to a very small diquark.

When probing this model further by confronting it with data on diquark fragmentation in neutrino-proton scattering, two of us found[4] that the $(ud)_0$ does not seem to break up when fragmenting into hadrons. We therefore assume that a spin-0 diquark always ends up inside a baryon (neglecting possible diquark-antidiquark bound states).

In our earlier study[3] of the influence of spin-0 $D\bar{D}$ pairs created in the color field of a fragmenting quark we found a best-fit value of only 225 MeV for the $(ud)_0$ mass and of 450 MeV for the $(us)_0$ and $(ds)_0$ masses. These are the values needed to explain the yield of slow baryons in e^+e^- annihilation. The 225 MeV also happens to be the value needed for reproducing the proton mass in the MIT bag model if the proton is treated as a bag with a massless u quark and a massive $(ud)_0$ diquark.

Before studying directly produced $D\bar{D}$ pairs, we can summarize the assumptions of importance for e^+e^- annihilation:

(i) The only diquarks of relevance are those with spin 0 and color $\underline{3}^*$. We neglect, for simplicity, the possibility of orbital or color excitations.

(ii) The scalar diquarks stay together and end up in baryons. Directly produced diquarks therefore give rise to leading baryons.

(iii) The size of the $(ud)_0$ corresponds to a size parameter of about 10 GeV^2 in the spacelike elastic form factor.

When extending the model to the direct process $e^+e^-\to D\bar{D}$, there is one feature that will dominate the predictions, namely, the appearance of heavier spin-0 diquarks at high energies. The diquarks are, in order of increasing mass, the $(ud)_0$, $(us)_0$, $(ds)_0$, $(uc)_0$, $(dc)_0$, $(sc)_0$, $(ub)_0$, $(db)_0$, $(sb)_0$, and $(cb)_0$.

The cross section for producing a certain $D\bar{D}$ pair in $e^+e^-\to D_i\bar{D}_i$ can be determined from the charge (squared), mass, and timelike form factor of the diquark D_i. As far as the charges are concerned, it is obvious that the charmed diquark $(uc)_0$ with $e_D{}^2=16e^2/9$ is of a particular interest.

To estimate the diquark masses, we start with the 225 MeV mentioned above for the $(ud)_0$. Then we assume that the heavier ones can be computed by just adding the *quark mass differences when one or more of the quarks in* the $(ud)_0$ are changed into a heavier quark. For the quark masses we take the values

$$m_u=m_d=0\ ,\quad m_s=225\text{ MeV}\ ,\quad m_c=1.5\text{ GeV}\ ,$$

and (2)

$$m_b=4.5\text{ GeV}\ .$$

This leads to the diquark masses given in Table I. Our final results do not depend much on the detailed diquark masses.

The crucial point for computing the diquark contribution to e^+e^- annihilation is naturally the timelike form factor of the heavier diquarks. The only independent piece of information we have is the empirical relation (1) for the spacelike form factor. It is not possible to continue this relation in a unique way to the timelike region $W^2=-Q^2>0$, since we do not know the exact dynamics in the diquark system. As long as there are no resonances in the $D\bar{D}$ system the form factor should fulfill $F(W^2)\le 1$, and then naturally $F(W^2)\to 0$ as $W\to\infty$.

There are reasons to believe that the falloff in $F(W^2)$ with rising W would be somewhat slower than that of $F(Q^2)$ with rising Q^2 in the spacelike region. First, this is true at intermediate W values for the straightforward analytic continuation of the expression in (1). Then one might argue that the mass parameter $M^2=10$ GeV^2 does not reflect the size of a "naked" $(ud)_0$ diquark, but rather of a $(ud)_0$ that is disturbed by the third quark in the nucleon. The $(ud)_0$ would then be even smaller in e^+e^- reactions than inside the nucleon. We try to take these points into account by using the simplest possible expression for the form factor of the diquark D_i:

$$F_i(W^2)=\begin{cases}1 \ \text{ at } \ 4m_i{}^2<W^2\le M_i{}^2\ ,\\ M_i{}^2/W^2 \ \text{ at } \ M_i{}^2<W^2\ .\end{cases} \tag{3}$$

Taking $M_i^2=10$ GeV^2 for the $(ud)_0$, it remains to find the $M_i{}^2$ values for the heavier diquarks. The $M_i{}^2$ is obviously related to the rms radius of the diquark through

$$M_i{}^2\propto\langle r^2\rangle_i{}^{-1}\ . \tag{4}$$

Finally, we make the simplifying assumption that this radius is inversely proportional to the reduced mass of the two-quark system, just like in a nonrelativistic Coulomb system. Hence,

$$(\langle r^2\rangle_i)^{1/2}\propto\mu^{-1}\ , \tag{5}$$

where

$$\mu=m_{q1}m_{q2}/(m_{q1}+m_{q2})\ , \tag{6}$$

and m_{q1} and m_{q2} are the masses of the two quarks in the diquark. Equations (5) and (6) do not apply to massless quarks. By giving the u and d quarks some small masses $(\ll m_s)$ we can, however, get the simple result

$$M_{us}{}^2=M_{ds}{}^2=M_{uc}{}^2=M_{dc}{}^2=M_{ub}{}^2=M_{db}{}^2=40\text{ GeV}^2\ . \tag{7}$$

This corresponds to the well-known result that a light-heavy two-particle Coulomb system has half the Bohr radius of a light-light system. Still, the reference to a nonrelativistic Coulomb system is naturally vague, and one could therefore regard Eq. (7) as nothing but a reasonable guess. As it will turn out, the only M^2 to be probed by the data is the $M_{uc}{}^2(=M_{dc}{}^2)$ due to the high charge of the $(uc)_0$.

TABLE I. The quark and diquark parameters used in Eqs. (4), (9), and (10). The particular choices are motivated in the text. Isospin symmetry has been assumed. The charge is in units of e.

Quarks and diquarks	Mass m_i (GeV)	Parameter $M_i{}^2$ (GeV^2)	(Charge)^2 summed
u,d	0		$\frac{5}{9}$
s	0.225		$\frac{1}{9}$
c	1.500		$\frac{4}{9}$
b	4.500		$\frac{1}{9}$
$(ud)_0$	0.225	10	$\frac{1}{9}$
$(us)_0,(ds)_0$	0.450	40	$\frac{5}{9}$
$(uc)_0,(dc)_0$	1.725	40	$\frac{17}{9}$
$(sc)_0$	1.950	$150-\infty$	$\frac{1}{9}$
$(ub)_0,(db)_0$	4.725	40	$\frac{5}{9}$
$(sb)_0$	4.950	$150-\infty$	$\frac{4}{9}$
$(cb)_0$	6.275	$150-\infty$	$\frac{1}{9}$

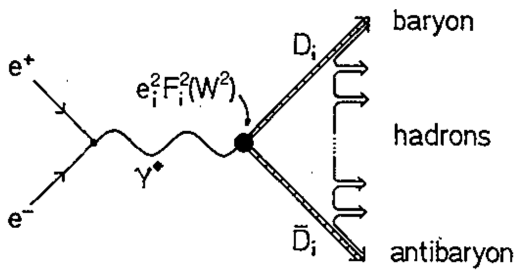

FIG. 1. The process $e^+e^- \to D_i\bar{D}_i$ in the one-photon approximation, followed by the diquark-antidiquark pair fragmenting into final-state hadrons. The photon-diquark coupling is determined by the diquark charge and timelike form factor.

With practically massless u and d quarks, all the diquarks that are not given in Eq. (7) will have higher M^2 values. There are no reasons to believe, though, that the $(sc)_0$, $(sb)_0$, and $(cb)_0$ diquarks are pointlike at, for instance, DESY PETRA energies $W \lesssim 40$ GeV, and we will therefore present results for a range of M^2 values for these diquarks. The parameters of the model are collected in Table I.

III. THE HADRONIC R FACTOR

Now we are ready to analyze the consequences of the diquark diagram in Fig. 1. The theoretical derivation of the cross section, in the one-photon approximation, for the process $e^+e^- \to D\bar{D}$ with scalar, pointlike D and $\bar{D}$ can be found in, for instance, Ref. 6. The contribution from the pair $D_i\bar{D}_i$ to the ratio

$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) \tag{8}$$

then reads, for extended scalars,

$$\Delta R_{D_i} = \tfrac{3}{4}e_i^2\left[1-\frac{4m_i^2}{W^2}\right]^{3/2} F_i^2(W^2)\,. \tag{9}$$

Here the factor 3 comes from summing over colors, the $\frac{1}{4}$ from the spin 0 and the $(1-4m_i^2/W^2)^{3/2}$ from the kinematic threshold effect. The corresponding formula for the quark process $e^+e^- \to q_i\bar{q}_i$ is

$$\Delta R_{q_i} = 3e_i^2\left[1-\frac{4m_i^2}{W^2}\right]^{1/2}\left[1+\frac{2m_i^2}{W^2}\right]. \tag{10}$$

The muon mass has been neglected in (9) and (10).

In Fig. 2 we plot R as the added contributions from (9) and (10) for the parameter values given in Table I. It can be seen that the agreement with data[10] is quite good. There are obviously three rather distinct regions in the energy W, and we discuss them separately below.

(i) The region $W \lesssim 3.5$ GeV. Here there are only light diquarks, which are not very influential. In addition, the data are not so accurate, and the fit is therefore less conclusive.

(ii) The region $3.5 \lesssim W \lesssim 15$ GeV. This is where the charmed diquarks appear and are predicted to contribute significantly to R, which explains the broad bump at $5 \lesssim W \lesssim 8$ GeV in the data of Ref. 11. Various other explanations of this structure were considered in Ref. 12, but none of those was found to be plausible. It should be noted, though, that Ref. 13 quotes some conflicting, but unpublished, data[14] as evidence against a bump in R. The difference between Refs. 11 and 14 is hard to analyze, since both data sets have been subject to several substantial systematic corrections, some of which are model dependent.

(iii) The region $W \gtrsim 15$ GeV. Here the quark contribu-

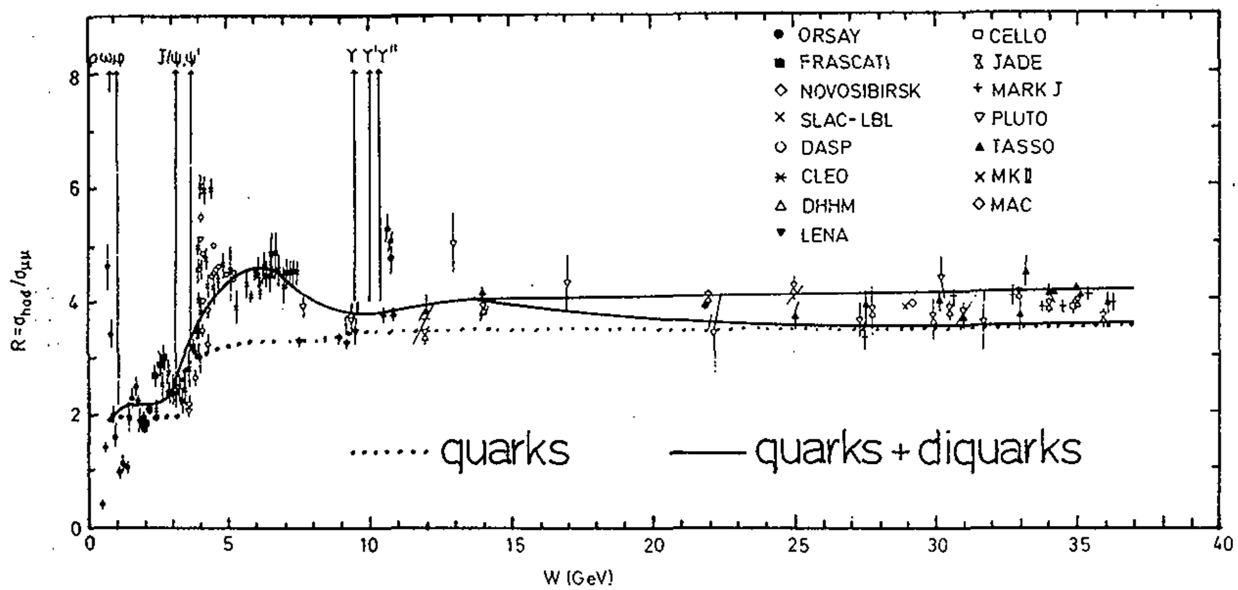

FIG. 2. The hadronic R factor defined by Eq. (8). The data points are taken from the collection in Ref. 10. The dotted line shows the contribution from quark-antiquark pairs, and the full lines display the added cross sections for quark-antiquark and diquark-antidiquark pairs; all according to Eqs. (9) and (10) with quark and diquark parameters taken from Table I. The two lines at energies $W > 14$ GeV show the range in R values consistent with the uncertainty in size parameters for the heaviest diquarks, as given in Table I. However, as argued in the text, data on, for instance, the two-jet thrust axis angular distribution give indirect support to the lower curve, representing a suppression of $D\bar{D}$ events at high W values.

tion underestimates the data with about 7% on the average. The error bars are, however, of the same order. It is therefore not meaningful to fit the "tail" of the almost pointlike diquark distributions to the data in this W region. On one hand, one could get a ΔR of around 10% by assuming that the $(sb)_0$ and $(cb)_0$ are pointlike all the way up to the highest energies, but it is, on the other hand, more likely that this excess in R is due to the events that show a three-jet structure. Presumably, the $D\bar{D}$ events can therefore be neglected at $W \gtrsim 15$ GeV. This naturally also applies to possible diquarks with the top quark.

IV. THE TWO-JET ANGULAR DISTRIBUTION

With the help of Eqs. (9) and (10) it is easy to compute the angular distribution of hadronic events as fitted to a two-jet structure. Taking θ as the cms angle between the beam and jet directions, the scalar diquark jets in $e^+e^- \to D\bar{D}$ are distributed like $1-\cos^2\theta$, while the quark jets from $e^+e^- \to q\bar{q}$ follow the familiar $1+\cos^2\theta$ distribution (neglecting a small $1-\cos^2\theta$ contribution just above the $q\bar{q}$ thresholds).

When fitting the data to a two-jet (thrust axis) angle distribution of the form

$$f_{2\,\text{jet}}(\theta) \propto 1+\alpha\cos^2\theta\ , \tag{11}$$

our model therefore predicts that the parameter α is related to the contribution from diquarks to the total hadronic cross section through the relation

$$\alpha = -3+4[1+\sigma(e^+e^- \to D\bar{D})/\sigma(e^+e^- \to \text{hadrons})]^{-1}\ . \tag{12}$$

We compute α from Eqs. (9) and (10) and display the result in Fig. 3 for W values below 15 GeV, together with the experimental results at 4.8–7.4 GeV (Refs. 11 and 15), 9–10 GeV (Ref. 16), and 10.5 GeV (Ref. 17). The error bars show statistical uncertainties only. The data points illustrated by unfilled circles at $W \leq 7.4$ GeV have been extracted by us from the data of Refs. 11 and 15.

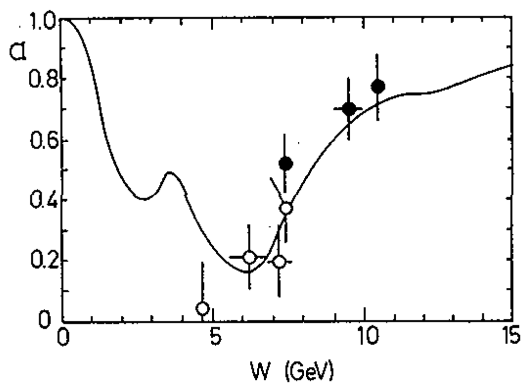

FIG. 3. The jet (thrust axis) angular distribution in two-jet events, when parametrized as $1+\alpha\cos^2\theta$. The filled data points are from Ref. 15 (7.4 GeV), Ref. 16 (9–10 GeV), and Ref. 17 (10.5 GeV), the unfilled ones from our analysis of the data of Refs. 11 and 15, as explained in the text. The curve shows the expectation from quark and diquark reactions through Eq. (12). Quark reactions alone would lead to $\alpha=1$ (neglecting kinematic mass corrections and gluon processes).

We have simply averaged the data[15] on the charged-hadron $\alpha(x)$ over $x=2p_{\text{had}}/W$ with the help of the data[11] on $d\sigma/dx$. We have not considered the smearing effect of quark and diquark fragmentation on the hadronic angular distribution. The filled circle at 7.4 GeV shows the result of a jet analysis with polarized beams in Ref. 15. That value should not be mixed up with the result "$\alpha=0.97\pm0.14$" presented in Ref. 15, which is achieved after a jet-model Monte Carlo simulation of $\alpha(x)$. Then 0.97 ± 0.14 is the limit of $\alpha(x)$ as $x\to1$. This represents the angular distribution of the partons that give rise to the fastest hadrons. These partons cannot be diquarks though, since a diquark always gives a massive baryon. At 7.4 GeV this rest mass effect limits x to values below around 0.75.

It is interesting to note that no significant deviation from $\alpha=1$ has been found above 15 GeV.[10] This seems inconsistent with the 10% admixture of diquarks needed for a best fit to R with two-jet diquark events only. Therefore we assume that all directly produced scalar $D\bar{D}$ pairs can be neglected beyond $W \approx 15$ GeV and that three-jet events are responsible for the 7% excess in R over the $q\bar{q}$ contribution. In a forthcoming publication we will discuss in detail a model for three-jet events suggested by us in Ref. 5. There we pointed out that if there is a substantial number of events like $e^+e^- \to D\bar{D}$, there should also be events with *unbound* two-quark systems in $e^+e^- \to qq\bar{D}$ and $e^+e^- \to \bar{q}\bar{q}D$. Such three-jet events would have some very distinct signatures and therefore deserve an analysis of their own. For the purpose of this work it suffices to note that a qq or $\bar{q}\bar{q}$ pair should have $J=1$ and a high effective mass in order to explain why these events survive at high energies but still do not give a $1-\cos^2\theta$ contribution when analyzed as two-jet events.

V. BARYON PRODUCTION

The yields and momentum spectra of produced baryons are ideal measures of the influence of diquarks in hadron-hadron, lepton-hadron, and e^+e^- reactions. The quark recombination model of Ref. 18, where three independent quarks join to form a baryon, can explain only a minor fraction of the baryon yields. In the improved quark-recombination scheme of Ref. 19 it has, however, been found that the size of the baryon yields is quite compatible with experimental observations. Detailed comparison with the data is, however, not possible due to the assumed flavor SU(3) symmetry of this approach. Many models contain, therefore, the assumption that baryons are created only when a $D\bar{D}$ pair appears during the fragmentation of a q or a $\bar{q}$. Such baryons are produced on the 8% level all over phase space. As mentioned earlier, the data here can be understood within our model[3] with the help of light $(ud)_0$, $(us)_0$, and $(ds)_0$ diquarks, and their antidiquarks, while the heavier diquarks are strongly suppressed. The contribution from the directly produced $D\bar{D}$ pairs, which are quite important in our approach, has some strikingly different properties, however.

In Ref. 8, Meyer tested a scheme for baryon production, which in our language would correspond to pointlike direct diquarks produced in 7.5% of the hadronic e^+e^-

reactions. Without specifying the possible quantum numbers or couplings of such diquarks, Meyer concluded that data on baryon production at 30–34 GeV e^+e^- energy are not accurate enough to establish such a 7.5% contribution. If our diquark parameters, as given in Table I, are correct, it would, however, be wiser to test this idea with accurate baryon data at energies of 4–8 GeV. A glance at Fig. 2 shows that here we expect up to 30% of the hadronic events to contain a baryon-antibaryon pair that has been created from a direct $D\bar{D}$ pair.

It is not possible to measure separately the contribution from direct diquarks, since they mix at all angles and momenta with the ones created from the color fields. Nevertheless, these two contributions to the yields of baryons have very different detailed features. We list these properties below.

(i) The number of baryons per event from vacuum $D\bar{D}$ pairs is roughly proportional to the number of produced pions all over phase space. Baryons from direct $D\bar{D}$ pairs appear, however, according to Eq. (9).

(ii) The quantum-number dependence is quite different in the two components. This is most apparent for charmed baryons. Charmed diquarks are too heavy to be created during the quark fragmentation, and therefore appear only as directly produced objects. The Λ_c, for example, could therefore be composed either of a direct c quark and a $(ud)_0$ diquark from the vacuum, or of a direct $(uc)_0$ or $(dc)_0$ and a d or u quark from the vacuum. In both cases the Λ_c is the leading particle in the jet, but the angle and W dependence of the two components will be very different.

(iii) The angle dependence is, in accordance with the previous discussion, roughly $1+\cos^2\theta$ for baryons from quark jets, but $1-\cos^2\theta$ from diquark jets.

(iv) The baryon-antibaryon correlation is also radically different in the two components. Vacuum $D\bar{D}$ pairs are created in the vacuum with little internal energy, and therefore come out in baryon-antibaryon pairs that are close in phase space. A direct $D\bar{D}$ pair, however, carries the full initial energy, and gives rise to a baryon and an antibaryon that are correlated back-to-back in angle.

(v) The baryon momentum distribution is more shifted toward high momenta for the direct component as compared to the indirect one, since a direct diquark always gives a leading baryon, while a vacuum diquark is slower on the average.

Detailed numerical predictions for the average number and momentum distribution of various baryons are outside the scope of this work, since they would require further assumptions about how quarks and diquarks fragment into hadrons. As has been demonstrated clearly enough in the literature, already the treatment of quark fragmentation requires quite complex Monte Carlo computer programs with numerous adjustable parameters. In addition, one would have to take into account that different baryons are detected by completely different techniques, with different sensitivities for, especially, the fast baryons that are of interest for probing the influence of the direct diquarks. The best case seems to be Λ production, because a large fraction of the Λ's should come from the crucial Λ_c decay.[20] The admixture of such decay products among the directly produced Λ's is, however, poorly known. Therefore, we list a few particularly clear *qualitative* trends that we expect in Λ production. These trends would, if confirmed, be practically impossible to understand in terms of vacuum $D\bar{D}$ pairs alone, and naturally also in terms of quark and gluon processes alone. The entries below are the same as in the previous list of properties.

(i), (ii) The number of Λ's per event should rise monotonously with W due to the vacuum $D\bar{D}$ pairs and the increase in phase space, but should on top of that have a clear structure in the energy region 4–8 GeV due to the decay of Λ_c's from direct diquarks. Such direct Λ_c's are expected to be about ten times as many at $W\approx 6$ GeV as from vacuum diquarks, which in turn implies that the mean number of Λ's per event is expected to be *several times* higher at 6 GeV than at somewhat higher energies.[21]

(iii) The angle distribution of Λ's at $W\approx 6$ GeV will consequently be dominated by the $1-\cos^2\theta$ component, so that the yield of Λ's will *increase* with outgoing angle and have a maximum at 90°. This feature will be particularly clear if measured for fast Λ's.

(iv) The $\Lambda-\bar{\Lambda}$ correlation at $W\approx 6$ GeV is expected to be dominated by the back-to-back effect in the direct $D\bar{D}$ pair. The distribution in the $\Lambda\bar{\Lambda}$ opening angle, $\theta_{\Lambda\bar{\Lambda}}$, will have a rather broad peak at $\theta_{\Lambda\bar{\Lambda}}=180°$ though, since the decays of the original back-to-back Λ_c and $\bar{\Lambda}_c$ lead to a smearing in the outgoing Λ and $\bar{\Lambda}$ directions.

(v) The Λ momentum distribution will be more extended toward higher momenta at $W\approx 6$ GeV than at other energies and than for other produced particles (such as pions).

It should be noted that current data on Λ production except those of Ref. 21, are taken at $W \gg 6$ GeV and therefore of no use for testing these predictions.

VI. CONCLUSIONS

We have shown that the present data from e^+e^- annihilation into hadrons leave room for the existence of very small spin-0 diquarks that can be produced directly from the virtual photon. When treated as elementary objects with a spatial extension described by a form factor, they are predicted to leave traces in several different connections. The main effect will be in the energy region $W=4$–8 GeV and come from the charmed diquarks $(uc)_0$, $(dc)_0$, and $(sc)_0$. Since they are expected to appear in baryons like the Λ_c (and Ξ_c), which decay frequently into Λ, the most crucial experimental test of the existence of small scalar diquarks would be to measure as many properties as possible of Λ production at $W\approx 6$ GeV.

Most of the trends predicted in the preceding paragraphs would, if confirmed, be hard to understand in terms of perturbative gluonic reactions, and would support the view that there are important nonperturbative effects in the form of diquark formation. That would naturally also have far-reaching consequences for the interpretation of other data within perturbative and nonperturbative QCD schemes.

ACKNOWLEDGMENT

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[21]The data of G. S. Abrams *et al.* [Phys. Rev. Lett. **44**, 10 (1980)] are consistent with such a structure but are too inaccurate at $W \gtrsim 6$ GeV to actually support the model. In addition, more than half the Λ's predicted by us should be absent in these data, because of the experimental cuts.

Paper III

LARGE-p_T PROTONS FROM CONSTITUENT DIQUARK SCATTERING

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Recent data from the CERN ISR on the fractional proton yield in pp collisions are explained within the Stockholm diquark model. Describing the proton as a $u(ud)_0$ system, the observed high magnitude and fall-off p_T, θ and $\sqrt{s}$ of the proton yield are natural consequences of constituent diquark elastic scattering. The p_T and θ dependence favour a value of around 10 GeV^2/c^2 for the size parameter in the diquark form factor, corresponding to a diquark rms radius of around 0.2 fm. This is consistent with earlier results of the model applied to deep inelastic lepton–nucleon scattering and e^+e^- annihilation.

1. Introduction. It is well known that the relatively high yield of protons in various high-energy processes is hard to understand within naive quark-parton models. This problem has been tackled mostly by introducing a finite probability for the initially struck quark to pick up a diquark during the fragmentation into hadrons. Such a creation of diquark–antidiquark pairs is conventionally assumed to result in production of baryon–antibaryon pairs on the level of 5–10% in comparison with pion production.

However, recent experimental results indicate that this mechanism is not sufficient for understanding the magnitude of proton production at high energies. The split field magnet group at the CERN ISR has found [1] that the fractional proton yield in 63 GeV pp collisions is not only high but also dependent on the transverse momentum, p_T, and CMS angle, θ, of the produced particle. Earlier, similar trends have been observed at Fermilab in pp and π^-p collisions in the 200–400 GeV/c range [2,3]. By comparing these data sets one can conclude that the fractional proton yield depends also on the collision energy, $\sqrt{s}$. Simultaneously, the fractional antiproton yield is an order of magnitude smaller and does not depend significantly on any of the variables p_T, θ and $\sqrt{s}$ in the quoted data sets. As pointed out by the CERN group [1], these findings cannot be understood within QCD-inspired quark fragmentation models, such as the Lund Monte Carlo or the Feynman–Field model, or within perturbative QCD, since the two processes that contribute to baryon production in these two classes of models, diquark–antidiquark pair production and gluon bremsstrahlung, respectively, would both lead to an equal number of protons and antiprotons. In addition, the Lund model obviously predicts a universal ratio, p/π^+, of proton to positive pion yields in all regions of phase space, reflecting only the fractional probability of creating a diquark–antidiquark pair instead of a quark–antiquark pair in the colour field from a struck quark. The ratio p/π^+ in fact exceeds unity at the lowest p_T values in ref. [1], and does not drop to its "natural" value of 5–10% until the squared momentum transfer from projectile to outgoing proton is well above a dozen GeV^2/c^2. This seems to exclude also explanations in terms of more exotic ("higher twist") quark processes like $q + q \to p + \bar{q}$ and $q + p \to p + q$. Both would be characterised by the proton form factor, which is strongly suppressive above a squared momentum transfer of 1 GeV^2/c^2. Neither can they explain why there are more protons than pions in some parts of phase space.

The aim of this letter is to show that all the features of the CERN ISR data on proton production can be reproduced within the Stockholm diquark model, developed by us and Jändel and Larsson earlier in a series of publications [4]. According to this mod-

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el, two quarks with unequal flavours can form a very small bound spin-0 system, a scalar diquark. No genuine spin-1 diquarks are assumed to exist. Therefore the proton is predominantly a $u(ud)_0$ system, with the $(ud)_0$ occupying only a few percent of the full proton volume, and with the gluon component being largely contained in the diquark. The high yield of protons from πp and pp collisions then comes about because of the possibility of diquark elastic scattering. The $(ud)_0$ diquark is elastically knocked out from a proton and thereafter fragments into a baryon. The observed fall-off in the fractional proton yield with p_T, θ and $\sqrt{s}$ is a natural consequence of the compositeness of the $(ud)_0$ diquark, which contributes a form factor in the scattering amplitude. This form factor represents the probability for the diquark to stay together during the scattering, and depends only on Q^2, the squared momentum transfer from the incoming to the scattered diquark. A slow fall-off with Q^2 means a small diquark, and our earlier analysis of lepton–nucleon scattering has led us to assume that the "break-point" Q^2 value in the $(ud)_0$ form factor is at least 10 GeV^2/c^2, which hints at a diquark radius smaller than 25% of that of the proton. If two-quark forces are strong and attractive enough to form such a small diquark, this non-perturbative QCD effect should appear in many other high-energy reactions. It has been speculated [5] that diquarks could be responsible for the bulk of QCD effects previously ascribed to perturbative gluonic reactions.

The Stockholm diquark model was used by Larsson [6] to explain the p/π^+ ratio in the Fermilab data from π^-p collisions quoted above. Here we must, however, make more specific assumptions, both inside and outside the domains of the original model, because the CERN ISR data are taken in kinematic regions where our previous fits of model parameters to lepton–proton scattering data do not help. A model very similar in spirit to ours has also been used by the CERN ISR group in a recent preprint [7] and shown to be in line with the data. Some basic assumptions about diquarks differ from our approach though, but this only strengthens our belief that the data give evidence for the existence of very small spin-0 diquarks inside nucleons, irrespective of the particular assumptions about the momentum distributions of the initial diquark, the fragmentation function of the outgoing diquark and the exact expression for the constituent scattering amplitude.

Some early attempts by other groups to analyse proton yields in terms of diquark scattering were quoted in ref. [6]. These models do not bear much resemblance to ours, and the old data were not detailed enough to pinpoint such important model parameters as the relative admixture, quantum numbers and radius of diquarks in nucleons.

2. The diquark in action. In order to derive proton yields from our model we need to specify quite a few quantities, some of which are already given in the model, while others have to be derived from independent data or fitted to the present CERN ISR data. The most relevant quantities are the following: (i) The diquark form factor; (ii) The momentum distributions of quarks and diquarks in the proton; (iii) The nature of the constituent subprocesses that give rise to pions and protons; (iv) The expressions for the constituent cross sections as functions of the kinematic variables; (v) The fragmentation functions for quarks and diquarks into pions and protons.

For the purpose of this work we assume that the strong form factor of the $(ud)_0$ diquark is identical to the electromagnetic form factor

$$F(Q^2) = M^2/(Q^2 + M^2)\,, \qquad (1)$$

used by us earlier [4]. Here we have found that $M^2 = 10\ \mathrm{GeV}^2/c^2$ (and, in fact, even values up to 20 GeV^2/c^2) fits well the data from deep-inelastic lepton–nucleon scattering. Such high M^2 values point to a diquark radius of 0.2 fm or smaller.

In ref. [4] we also found that the momentum distribution of the $(ud)_0$ is fairly similar in shape to that of a u quark at Bjorken x values of $0.25 < x < 0.75$. We therefore assume here that $x\mathrm{D}(x) = x\mathrm{u_v}(x)$ for all x, D being the $(ud)_0$ and $\mathrm{u_v}$ the single u quark. In a more recent analysis [8] we, in turn, argued that the single u quark momentum distribution can be derived by taking the difference between proton and neutron deep inelastic structure functions $F_2^{\mu p}$ and $F_2^{\mu n}$. The data of ref. [9] fit well to the parametrisation

$$\begin{aligned} x\mathrm{D}(x) &= x\mathrm{u_v}(x) \\ &= 0.3(0.87 - x) \quad \text{at } 0.25 < x < 0.87\,, \\ &= 0 \quad \text{at } 0.87 \leqslant x\,. \end{aligned} \qquad (2)$$

The region $x < 0.25$ is not relevant for the present analysis. It is also necessary to specify the momentum distributions $xu_D(x)$ and $xd_D(x)$ of the u and d quarks inside the $(ud)_0$, because the diquark is gradually dissolved into its two constituents as Q^2 increases. The vanishing of the diquark contribution as $F^2(Q^2)D(x)$ is therefore accompanied by a corresponding enhancement of the contribution from its quarks as $(1-F^2)q_D$. We assume isospin symmetry, $u_D \approx d_D$, and adopt the well-known parametrisation

$$xd(x) = xd_D(x) = xu_D(x) = 1.23\sqrt{x}(1-x)^4 , \tag{3}$$

of the d quark momentum distribution. Finally, we neglect contributions from sea quarks and gluons.

When it comes to keeping track of all the subprocesses that can give rise to protons and pions we will make a few simplifying assumptions. First, we split up only one of the initial protons in quarks and diquarks, while treating the other as an effective target with the mean constituent momentum distribution

$$xq_{eff}(x) \propto \sqrt{x}(1-x)^3 . \tag{4}$$

This ignorance of the detailed structure of the target is motivated by the fact that we will study only the fractional proton yields from scattered quarks and diquarks in the projectile, and they turn out not to depend much on target properties. In addition, we will choose a phenomenological constituent cross section that is known to reproduce pion yields successfully when the target is described by eq. (4). Secondly, we assume that only leading hadrons from the fragmenting quarks and diquarks contribute to the ratio of protons to positive pions at large p_T values. Then we need to consider only three processes: (i) a scattered $(ud)_0$ fragmenting to a proton; (ii) a scattered u quark giving a leading π^+; (iii) a u quark giving a leading proton. The latter case happens when the u quark picks up a diquark from a created $D\bar{D}$ pair. We assume that this occurs for 5% of the scattered u quarks. As the CERN ISR data are presented as the ratio of protons to all positive hadrons, we also have to consider K^+ production and therefore assume that $K^+/\pi^+ \equiv 0.5$ in all the phase space of relevance for this analysis. This conjecture has support from the 90° Fermilab data [2] as well as from the 45° CERN ISR data [1], while the influence of kaons in the 13° and 20° CERN ISR data is unknown.

All constituent elastic cross sections are assumed to be of the empirical form suggested by Field and Feynman [10] for quark–quark elastic scattering:

$$d\hat{\sigma}/d\hat{t} \propto -1/\hat{s}\hat{t}^3 , \tag{5}$$

where $\hat{s}$ and $\hat{t}$ are the Mandelstam variables for the constituent process. This choice makes certain that we reproduce the measured pion spectrum with the simplified eq. (4) for the target substructure. Other, more QCD inspired, choices [11] give practically the same proton-to-pion ratios. The rapid fall-off with $\hat{t}$ guarantees that we can neglect the quarks and diquarks from the "target" proton that are backscattered to $180° - \theta$, since $\theta = 13°$, 20° and 45° in the data of ref. [1].

Also the function describing the fragmentation, $D \to p$, of a diquark to a proton is a priori unknown, i.e. cannot be taken from some independent data (backward protons from $\bar{\nu}p \to pX$, for instance, could come also from the non-diquark ud combination). Therefore, we adopt for the diquark fragmentation function the formula used by Peterson et al. [12] for describing the fragmentation of heavy quarks:

$$D^{p}_{(ud)_0}(z) = Nz^{-1}[1 - 1/z - \epsilon/(1-z)]^{-2} , \tag{6}$$

where ϵ is a parameter that was supposed in ref. [12] to be inversely proportional to the squared quark mass, and where N is a constant which can, in principle, be used to normalise the fragmentation function. We prefer, however, to keep both ϵ and N as free parameters to be fitted to the data, since z is restricted to $z \gtrsim 0.3$ in the data of ref. [1], and we do not want to commit ourselves to formula (6) also for smaller z values. For quarks fragmenting to pions we again follow ref. [12] and use

$$D^{\pi^+}_{u}(z) = 0.95(1-z)^2/z , \tag{7}$$

and consequently

$$D^{p}_{u}(z) = 0.05(1-z)^2/z , \tag{8}$$

in line with our assumption of a 5% probability of $D\bar{D}$ production in the fragmentation chain. As we study only fractional yields and keep N in eq. (6) free, we need not care about the absolute normalisation of (7) and (8).

The inclusive yield of hadron C from the subprocess $ab \to cX$ is

$$E\frac{d\sigma}{d^3p} = \int_{x_a^{min}}^{1} dx_a \int_{x_b^{min}}^{1} dx_b G_{A\to a}(x_a) G_{B\to b}(x_b) \times D_c^C(z)(\pi z)^{-1} d\hat{\sigma}/d\hat{t} . \tag{9}$$

Here a is a diquark or quark from the "projectile" proton and $G_{A\to a}(x_a)$ the functions $D(x_a)$, $u_v(x_a)$ or $u_D(x_a)$ given by eqs. (2), (3), $D(x_a)$ accompanied by the squared form factor $F^2(Q^2)$ and $u_D(x_a)$ by the complementary $1 - F^2$. Similarly, b is the mean constituent of the "target" proton, and $G_{B\to b}(x_b)$ is hence given by $q_{\text{eff}}(x_b)$ in eq. (4). $D_c^C(z)$ is taken from any of eqs. (6) to (8) and $d\hat{\sigma}/d\hat{t}$ from eq. (5). Finally, eq. (9) is summed over $a = (ud)_0$, u_v and u_D for proton production and over $a = u_v$ and u_D for pion production. The kinematic variables obey the following relations:

$$x_a^{\min} = x_T \cot \tfrac{1}{2}\theta/(2 - x_T \tan \tfrac{1}{2}\theta)\,, \tag{10}$$

$$x_b^{\min} = x_a x_T \tan \tfrac{1}{2}\theta/(2x_a - x_T \cot \tfrac{1}{2}\theta)\,. \tag{11}$$

$$z = \tfrac{1}{2}x_T(x_a^{-1}\cot \tfrac{1}{2}\theta + x_b^{-1}\tan \tfrac{1}{2}\theta)\,, \tag{12}$$

$$-\hat{t} = Q^2 = (s x_a x_T/2z)\tan \tfrac{1}{2}\theta\,, \tag{13}$$

where $x_T = 2p_T/\sqrt{s}$.

3. Results and discussion. When confronting eq. (9) with the data on the fractional proton yields we have manipulated only the fragmentation function for diquarks in eq. (6) by testing various values of the parameters ϵ and N. In all calculations they are, however, combined as to fit the fractional proton yield at $\theta = 13°$ and $p_T = 2$ GeV/c. With this restriction we test the set of fragmentation functions within the shaded area in fig. 1. The resulting fits to data are shown in fig. 2 for three different values of the crucial parameter M^2 in the diquark form factor of eq. (1). It can be seen that $M^2 = 10$ GeV$^2/c^2$ gives a good fit to the data, while 5 and 20 GeV$^2/c^2$ cannot reproduce the θ dependence from 13° to 20°. At 45° the form factor suppresses the diquark scattering so that the protons come mostly from quark fragmentation, which explains why the fit is less sensitive to M^2 here than at 20°.

Before concluding that the Stockholm diquark model is successful also in reproducing these large-p_T data, it is important to analyse the sensitivity of the fits to the various extra assumptions made necessary by our lack of knowledge of the important diquark fragmentation function and elastic cross section. First, it should be noted that these two quantities appear together in eq. (9), which means that even if $d\hat{\sigma}/d\hat{t}$ would differ drastically from subprocess to subprocess, we would only find another best-fit fragmentation function for the diquark, while the quality of the fit would be about the same. Secondly, we have found that the particular choice of diquark fragmentation function is important only for the absolute normalisation of the fractional proton yield, i.e. for the fit to one single data point ($\theta = 13°$, $p_T = 2$ GeV/c, say). In addition, it seems that the most crucial feature of the diquark fragmentation function is its value in the region $z \approx 0.7$ in comparison with that of the quark fragmentation function.

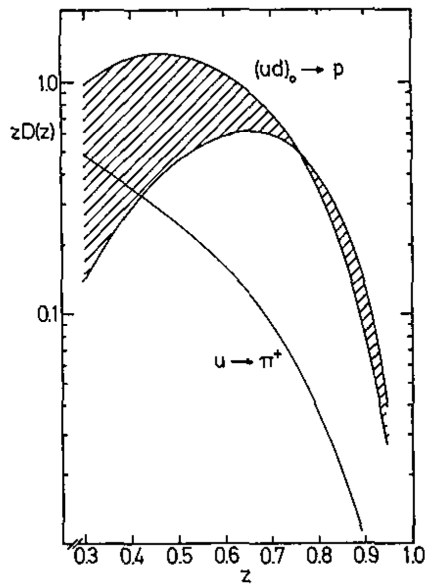

Fig. 1. The quark and diquark fragmentation functions used in the analysis, as given in eqs. (6) and (7). The normalisation is arbitrary, since it is unimportant for the fractional hadron yields. The shaded area shows the range of diquark fragmentation functions used for the parameter value $M^2 = 10$ GeV$^2/c^2$ in the diquark form factor, eq. (1). The extreme values of the parameters N and ϵ in eq. (6) are (17.5, 1.35) and (1.1, 0.28). For all diquark fragmentation functions they are combined as to fit the data point at $p_T \approx 2$ GeV/c, $\theta = 13°$. The same procedure for $M^2 = 5$ and 20 GeV$^2/c^2$ gives sets of fragmentation functions that are similar in shape but different in magnitude from the one shown for 10 GeV$^2/c^2$. The variable z is the fraction of the constituent momentum carried by the detected hadron.

Consequently, the interesting fall-off in the fractional proton yield with p_T and θ is sensitive practically only to the diquark form factor, and therefore gives a good measure of the size parameter M^2. The form factor naturally also influences the absolute yield of protons, but that effect alone cannot be distinguished from that of the fragmentation function.

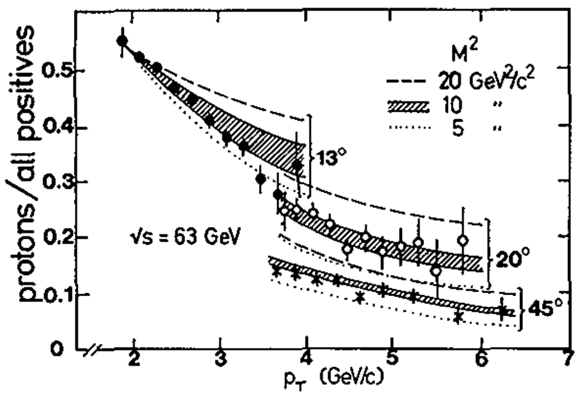

Fig. 2. The inclusive yield of protons relative to that of all positively charged hadrons as a function of p_T, the hadron transverse momentum, for three different CMS production angles, and at a CMS collision energy of 63 GeV. The experimental data are from the CERN ISR [1], and the lines are the results of the Stockholm diquark model. The shaded area for the parameter value $M^2 = 10$ GeV$^2/c^2$ in the diquark form factor of eq. (1) corresponds to the set of diquark fragmentation functions given by the shaded area in fragmentation functions given by the shaded area in fig. 1. For $M^2 = 5$ and 20 GeV$^2/c^2$ these areas have been indicated only by their centre lines [when using all fragmentation functions with $0.28 \leqslant \epsilon \leqslant 1.35$ in eq. (6) and taking N as to fit the leftmost data point].

An independent test of our choice of form factor is provided by the $\sqrt{s}$ dependence from Fermilab to CERN ISR data. The 200–400 GeV/c data on the ratio p/π^+ [2,3] are well fitted by the present formalism for $p_T \geqslant 4$ GeV/c and $\theta = 90$–$113°$. At $p_T \leqslant 3$ GeV/c we overestimate the data by 20–40%, which could be due to an incorrect choice of K^+/π^+, or to the fact that the Fermilab data probe other z values in the fragmentation function, where eq. (6) perhaps does not work so well. The nice fits in ref. [6] with equal fragmentation functions for quarks and diquarks support this guess.

By ignoring the substructure of one of the protons, we might bias the results. The diquark–diquark scattering should, for instance, be isotropic instead of following the $1/\hat{t}^3$ fall-off. However, that would be almost compensated by the extra form factor $\sim 1/\hat{t}^2$ from the "target" diquark, and the best-fit proton yields would stay almost the same as before.

Our claim that the data provide a good measure of the size of the $(ud)_0$ diquark in the proton is further supported by the fact that the CERN ISR group [7] finds practically the same best-fit values of M^2 (10–20 GeV$^2/c^2$) in an analysis that is completely different from ours in details, but shares the view that there exists a $(ud)_0$ diquark with a form factor given by (1). It is encouraging to find that this parameter is, in turn, consistent with the one we have already found to fit other data, such as the scale-breaking in deep-inelastic lepton–nucleon scattering [4]. Unlike the situation for those data, there seems, however, not to be any realistic alternative explanation in terms of conventional perturbative quantum chromodynamics and its gluonic processes, for the high yield of protons at high p_T. We consider this fact to be a strong support for our view [4,5] that the non-perturbative phenomenon of diquark formation might be responsible for many of the data trends hitherto attributed to perturbative gluon reactions.

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Paper IV

NEW IDEAS ON THE PROTON–NEUTRON DIFFERENCES IN DEEP INELASTIC STRUCTURE FUNCTIONS

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It is shown how future data from deep inelastic scattering of charged leptons on protons and neutrons can be used to discriminate between the two main possible scale-breaking QCD effects – perturbative gluon processes and non-perturbative diquark formation. Current data from the Stanford Linear Accelerator Center and from CERN on the structure function combinations $F_2^p - F_2^n$, F_2^n/F_2^p and σ_L/σ_T are in line with both the Stockholm diquark model and perturbative QCD, considering the low statistics as well as the possibility of a substantial normalisation error between the two data sets. This calls for a new high-statistics experiment on a deuterium target before any definite conclusions can be made.

1. Introduction. Are quark interactions dominated by perturbative or non-perturbative quantum chromodynamical phenomena? We have addressed this question in a series of publications [1–9], in order to demonstrate that the particular non-perturbative QCD phenomenon of diquark formation deserves special attention. From our analyses of various high-energy data it seems as if a strong bound-state QCD effect in the two-quark dynamics might explain many of the trends in high-energy data that have hitherto been attributed to "perturbative QCD effects". Those data leave it as an open question whether the commonly accepted "best-fit" value of the perturbative QCD expansion parameter α_s is correct, or whether diquarks are so important that only a smaller α_s would be reconcilable with the data.

Naturally, our analyses of diquark effects aim at finding reactions and data where these can be clearly discriminated from perturbative gluon effects. Although giving many predictions along these lines [2–5], we have so far only found processes where *either* each explanation is equally compatible with the data *or* perturbative QCD does not have much to say. Examples of the latter are the fragmentation of two-quark systems in neutrino–nucleon scattering [6], the rates of large-p_T protons in hadronic collisions [8, 9], and the detailed hadronic cross section in e^+e^- annihilation [5].

In this letter, we present a simple analysis of deep inelastic scattering of charged leptons on protons and neutrons. It shows that our diquark model gives predictions for the differences between proton and neutron targets that are quite opposite to those of perturbative QCD in the leading-log approximation. The published data from the Stanford Linear Accelerator Center (SLAC) [10] and the CERN European Muon Collaboration (EMC) [11] are in line with both models. Virtually, there are some aspects of the SLAC data that fit better to perturbative QCD, while a comparison between SLAC and QCD results seems to favour our model. However, as there remains the problems of large statistical errors in both experiments and of possible systematic normalisation errors between the two experiments, those observations cannot really be claimed to be in favour of either model at present. Hopefully, a more definite conclusion can be drawn in the light of future high-statistics deuterium data from the CERN EM Collaboration.

The observables we have in mind are (i) the difference, $F_2^p - F_2^n$, of proton and neutron structure functions, (ii) the ratio F_2^n/F_2^p, and (iii) the ratio $R = \sigma_L/\sigma_T$ of the cross sections for longitudinally to transversely polarised virtual photons, taken on both proton and deuteron targets; all as functions of the Bjorken variable $x = Q^2/(2m_p\nu)$ and the squared momentum transfer, Q^2 (ν is the energy transfer).

For the perturbative QCD fits we adopt the formalism of Duke and Owens [12]. These authors derive quark and gluon momentum distributions by integrating the Altarelli–Parisi equations [13] and fitting all parameters to the world data on various processes where perturbative QCD corrections are supposed to be dominant. It turns out that the results of relevance to this analysis depend more on the qualitative features of perturbative QCD than on the actual values of all these parameters.

For the diquark calculations we use the Stockholm diquark model, formulated most recently in ref. [9]. The main assumption is that the nucleon is predominantly in a state of a very small spin-0 $(ud)_0$ diquark plus a single quark. We have estimated the $(ud)_0$ radius to be of the order of 0.2 fm [1,9]. This value is represented by a diquark form factor, $F(Q^2) = M^2/(M^2 + Q^2)$, the square of which is the probability that the diquark interacts collectively with a virtual photon of four-momentum squared $q^2 = -Q^2$. The best-fit value of M^2 from various processes turns out to be around 10 GeV^2, which is remarkably pointlike, and motivates our conjecture that diquarks might be more important than perturbative gluons in many experiments. Other diquark models have been less drastic in this respect, with fewer and more loosely bound diquarks. See, for instance, ref. [14] and references therein.

At $Q^2 \gg 10$ GeV^2 the diquark is resolved into its constituents, a u and a d quark, and this must be taken into account in a way that also avoids double-counting at low Q^2. We therefore postulate that three sources contribute to the proton structure function:

(i) scattering on the single valence quark, u_v,

(ii) scattering on the diquark, D, (suppressed by the form factor squared),

(iii) scattering on one of the quarks, q_D, inside the diquark, (suppressed by the complement, $1 - F^2$, of the squared form factor).

This is a more precise definition of the model than the one used in our first phenomenological fits [1,2], where it was assumed that a nucleon contains some fractional numbers of diquarks and quarks, all to be extracted from data.

This formulation of the model is limited to the Q^2 regime well above 2 GeV^2, since at lower Q^2 there is a disturbance from "accidental" pairs of quarks that interact collectively with the photon (just as there is a contribution from the whole nucleon at even lower Q^2). Such accidental quark pairs consist of the single quark plus a quark in the genuine $(ud)_0$ diquark. They could be a ud in spin 0 or 1, or a uu in spin 1, but would have a mean size of 0.5–1 fm, and should therefore vanish at $Q^2 \gg 2$ GeV^2 [2].

2. The proton–neutron difference $F_2^p - F_2^n$. Here we will make some almost trivial observations concerning the Q^2 dependence of $F_2^p - F_2^n$ in the two models, as well as two non-trivial observations concerning the x dependence.

In perturbative QCD, one adopts the well-known result from the quark–parton model that

$$3(F_2^p - F_2^n) = x u_{val}(x, Q^2) - x d_{val}(x, Q^2)\,, \qquad (1)$$

the modification being that the valence quark distributions u_{val} and d_{val} depend not only on x but also on Q^2. We have assumed isospin symmetry when going from proton to neutron, and internal isospin symmetry among the sea quarks in either nucleon. As perturbative QCD has flavour independent corrections, which in turn almost factorise in $q(x, Q^2)$, one sees from (1) that $F_2^p - F_2^n$ is expected to have roughly the same Q^2

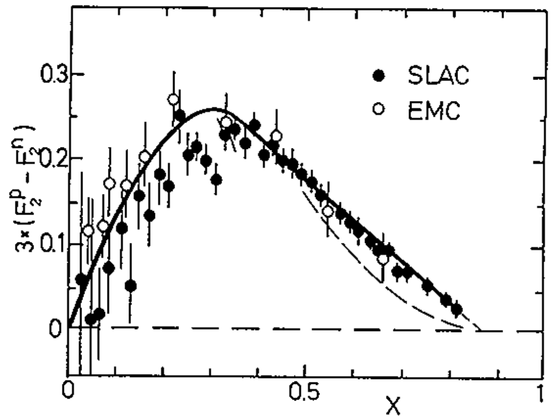

Fig. 1. The quantity $3(F_2^p - F_2^n)$ versus the Bjorken scaling variable x. The data points are from SLAC [10], with mean squared momentum transfer $\langle Q^2 \rangle = 2$–15 GeV^2, and from the CERN EMC experiment [11], with $\langle Q^2 \rangle = 9$–70 GeV^2. The error bars are statistical only. Our diquark model predicts the SLAC and EMC data to be equal at $x \gtrsim 0.3$. The full line is the parametrisation used in the further analysis, and the dashed line at $x > 0.5$ is the prediction for the EMC results from leading-log perturbative QCD in the parametrisation of ref. [12], which gives a good fit to the SLAC data.

dependence as the structure functions taken by themselves, except possibly at $x \lesssim 0.3$, where the sea contribution to F_2 might have a Q^2 dependence different from that of valence quarks. With the parameters of Duke and Owens [12], eq. (1) gives the result shown in fig. 1. The fit to the SLAC data is excellent, and therefore not plotted, while the corresponding solution, taken at the Q^2 values of the EMC data, runs through the lower parts of the statistical error bars at $x \gtrsim 0.5$.

A more precise analysis of the bulk of the (Q^2-binned) data on $F_2^p - F_2^n$ within perturbative QCD has been presented in ref. [15]. A good fit is obtained with the conventional value of $\Lambda_{\overline{MS}} \approx 0.3$ for the QCD scale-breaking parameter, but only after multiplying the (high-Q^2) EMC data with an ad hoc normalisation factor of 0.7 compared to the (low-Q^2) SLAC data. Taking the EMC data at face value would lead to a lower best-fit value of $\Lambda_{\overline{MS}}$, which would even by consistent with zero on the one-standard-deviation level, as can be seen by an inspection of fig. 1 in ref. [15]. This is naturally the same qualitative effect as on the one-error-bar level in the Q^2-integrated data of our fig. 1. An interesting, recent analysis that could be relevant for this problem has been given in ref. [16]. There the experimental group presents best-fit values of Λ_{LO} to its data on high-Q^2 neutrino–hydrogen and neutrino–deuterium collisions. It turns out that Λ_{LO} is considerably smaller, and consistent with zero, when deduced from the (high-Q^2) data on $F_2^n - F_2^p$ than when taken from the structure functions separately. Typical values are $\Lambda_{LO}^{n-p} = 55\,^{+120}_{-55}$ MeV and $68\,^{+120}_{-68}$ MeV in two different perturbative QCD schemes. Naturally, also these results need to be supported by higher statistics to be conclusive here.

In the Stockholm diquark model $F_2^p - F_2^n$ becomes trivially Q^2 *independent* since it fulfils

$$3(F_2^p - F_2^n) = x u_v(x)\,, \tag{2}$$

where u_v is the "non-diquark" single u quark in the proton. This is so because all other contributions cancal due to isospin symmetry; the diquark and its two constituent quarks, since they are the same in protons and neutrons, and the sea by the assumption of local isospin symmetry. As we have attributed all of the Q^2 dependence to the diquark form factor, the single quark distribution u_v *is a function only of* x. This prediction is clearly consistent with the data in fig. 1.

The thick curve shows the choice of $xu_v(x)$ that we will use as input for the further analysis of F_2^n/F_2^p and σ_L/σ_T below.

Two other data trends are in line with our model; the practically *linear* drop with x in fig. 1 for $x > 0.4$ and the *vanishing* at $x \approx 0.9$ of such a linear extrapolation of the data. A linear drop with x of $xu_v(x)$ is expected from dimensional counting rules [17] within our model, since the single u quark has only *one* effective spectator. This means that the particular u quark probed by $F_2^p - F_2^n$ is confined to the proton by exchanging gluons with only one effective coloured object, the diquark, and not with two. Similar conclusions about the effective colour interaction in nucleons have been drawn recently by two groups, from field-theoretic arguments. Betman and Laperashvili [18] derive bound diquarks from a theory by 't Hooft [19] for the instanton contribution to four-quark interactions, and Olivier [20] makes a case for diquarks from a string model for mesons and baryons.

The vanishing at $x = x_{max}$ of a linear extrapolation for $F_2^p - F_2^n$ is expected in our model since the $(ud)_0$ carries a *rest mass*, i.e. an energy which is effectively inaccessible to the single quark u_v. Unlike many other rest mass effects, this one does not vanish as $Q^2 \to \infty$. By studying the kinematics of the lepton–quark scattering in the infinite momentum frame one gets the simple result

$$x_{max} = 1 - m_D^2/m_p^2\,, \tag{3}$$

where m_D is the rest mass of the $(ud)_0$ when treated as a pointlike particle. Therefore $x_{max} \approx 0.9$ corresponds to a value $m_D \approx 300$ MeV. This mass value fits well the 330 MeV obtained by Betman and Laperashvili [18] and the 225–300 MeV we obtained earlier from an analysis [4] of baryon production in e^+e^- annihilation. It should not, however, be mixed up with constituent masses for diquarks as deduced from potential models for baryons. Such models would rather need the masses of u and d to be 400 MeV, to fit the mass of the diquark-free Δ, and the mass of the $(ud)_0$ to be 500 MeV, to fit the proton mass [‡1].

‡1 It is interesting to note that one can predict the diquark radius from classical considerations [21]: When two objects of mass 400 MeV bind into a system of mass 500 MeV, *the radius becomes around 0.25 fm, in agreement* with our determination of the diquark size from the mass parameter in the form factor.

The data in fig. 1 can also be used to set limits on diquark models in general. The large-p_T data from the CERN ISR that we analysed recently [9] require very small diquarks, with M^2 of the order of 10 GeV^2, but since in these processes the charges are not probed, there remains the possibility of small $(uu)_1$ and $(ud)_1$ diquarks in spin 1. An admixture of $(uu)_1$ in the proton would not, however, be cancelled by the corresponding $(dd)_1$ in the neutron when taking $F_2^p - F_2^n$. Therefore, the data can be used to set limits on the influence of (uu) diquarks in the proton wave function. It turns out that the equality of the SLAC and EMC data on the one-standard-deviation level can allow for at most 3–4% (uu) diquarks with $M^2 = 10$ GeV^2 at $x = 0.4$–0.7, and at most 6–11% if $M^2 = 3$ GeV^2. This seems to rule out spin-1 diquarks as a relevant source of large-p_T protons in pp collisions.

Before closing this section, we should make a comment on the possible Q^2 dependence in fig. 1 at $x < 0.3$. The tendency for the EMC data to fall about 0.02 units above the SLAC data here is consistent with a similar trend in F_2^n/F_2^p (see below) and in F_2^{Fe}/F_2^D (the so-called EMC effect [22]). These differences between SLAC and EMC results could arise from deviations between the two data sets on deuterium, which in turn could have to do with various Q^2 dependent effects, such as kinematic mass effects, diquark effects in the sea, an "EMC effect" already in the deuteron, or systematic experimental errors not fully understood. Since our main interest is in the valence region $x > 0.3$, we will not speculate further about this possible effect.

3. The ratio F_2^n/F_2^p. For this ratio the predictions from perturbative QCD and from the Stockholm diquark model are opposite to those for $F_2^p - F_2^n$: Perturbative QCD predicts an almost Q^2 *independent* ratio, while our model gives a significant drop with Q^2 at $x > 0.4$.

In the approach of Duke and Owens [12] it is not apparent from the parametrisations of $x(u_{val} + d_{val})$ and xd_{val} that the perturbative QCD corrections vanish in the ratio of d/u because of flavour independence. After performing the actual calculations it turns out that F_2^n/F_2^p indeed is predicted to drop by as little as around 0.01 units at $x > 0.4$ when going from the Q^2 values of the SLAC data to those of the EMC data. However, the predicted magnitude of F_2^n/F_2^p under-

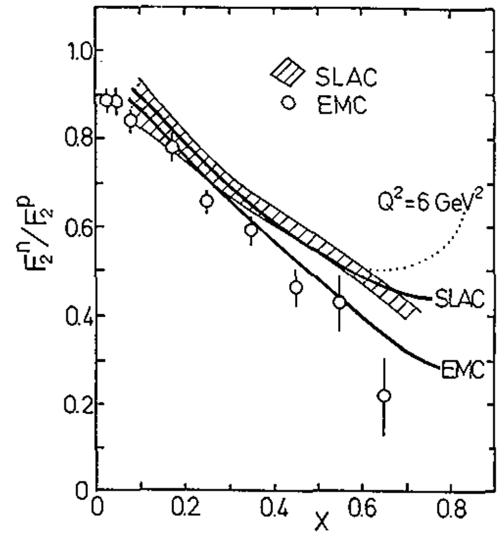

Fig. 2. The ratio F_2^n/F_2^p versus Bjorken x. The band illustrates the SLAC data and the points are the EMC data [11], both sets with statistical errors only. The full lines are the results of our diquark model calculated at the respective $\langle Q^2 \rangle$ values of the two data sets at each x. The perturbative-QCD parametrisation of ref. [12] (not shown) gives a poor fit to both data sets, and, in addition, predicts them to differ from each other by no more than 0.01 units. The leading-log parametrisation of ref. [23] fits the SLAC data, but gives, by construction, no difference between the SLAC and EMC cases.

estimates the data substantially at $x > 0.3$, so we have not plotted the result together with the data in fig. 2. The failure here could be due to an incorrect choice of the sea distribution, which should perhaps extend to higher x than suggested by Duke and Owens. In the leading-log parametrisation of ref. [23], a better fit is achieved to the SLAC data, but there the Q^2 independence of d/u is explicitly assumed.

In our model the proton structure function becomes

$$F_2^p = \tfrac{4}{9}xu_v(x) + \tfrac{1}{9}xD(x)F^2(Q^2) + \tfrac{5}{9}xq_D(x)\{1 - F^2(Q^2)\} + xS(x)\,, \qquad (4)$$

and correspondingly for the neutron:

$$F_2^n = \tfrac{1}{9}xu_v(x) + \tfrac{1}{9}xD(x)F^2(Q^2) + \tfrac{5}{9}xq_D(x)\{1 - F^2(Q^2)\} + xS(x)\,, \qquad (5)$$

in accordance with the model assumptions stated in the introduction. Here $D(x)$, $q_D(x)$ and $S(x)$ denote the distributions of the $(ud)_0$ diquark, any of the quarks inside the diquark, and the sea quarks (the latter with their charges), respectively. For the sea we choose $xS(x) = 0.3(1-x)^4$, which well fits the neutrino data [24], and for $xu_v(x)$ we take the curve in fig. 1. The distributions $xD(x)$ and $xq_D(x)$ cannot be directly extracted from some independent data. We suggest that $xD(x) = Ax^{\alpha}(1-x)^{1.5}$, with α to be adjusted to the data. This corresponds to the dimensional scaling law with one spectator and a helicity difference of 1/2 between the proton and the diquark. A nice fit to the data is obtained with $\alpha = 1$, which fixes A to be 2.5. Finally, we choose $xq_D(x) = B\sqrt{x}(1-x)^{\beta}$ and regard β as another adjustable parameter. One knows from conventional quark model fits to F_2 that the d quark is well described with $\beta \approx 4$ at $\langle Q^2 \rangle = 8$ GeV^2. In our model $xq_D(x)$ is identical to the quark model $xd(x)$ when $Q^2 \to \infty$. We therefore expect $\beta \gtrsim 4$, and find that $\beta = 4.5$ gives a good fit to the data. Hence we can summarise:

$$xD(x) = 2.5x(1-x)^{1.5}, \quad xq_D(x) = 1.2\sqrt{x}(1-x)^{4.5},$$
$$xS(x) = 0.3(1-x)^4. \tag{6}$$

The data on F_2^n/F_2^p turn out to restrict the possible choices for the distributions more severely than the pp data treated in ref. [9]. This is due to the fact that the latter analysis required as input also the a priori unknown diquark fragmentation function and strong scattering cross section. This is why the quark and diquark distributions here are somewhat different, and hopefully more accurate, than those used in ref. [9].

The results from eqs. (4)–(6) are shown together with the data on F_2^n/F_2^p in fig. 2. The "SLAC" and "EMC" lines are calculated for the respective $\langle Q^2 \rangle$ values at each x. We also show the prediction for a fixed Q^2 value of 6 GeV^2.

When it comes to comparing the two different predictions with the data one finds that the SLAC results, taken by themselves, are not accurate enough at $Q^2 \geqslant 2$ GeV^2 to reveal any consistent Q^2 dependence at fixed x values. However, the drop in F_2^n/F_2^p from SLAC to EMC is of the order of two EMC error bars at $x > 0.4$. Our model gives a satisfactory fit to this trend, but there remains the problem of possible normalisation errors in the deuterium data alone, which could explain the whole trend. It should be noted that there are preliminary EMC data (see, e.g., ref. [25]), which together with the data in fig. 2 would make the right-most data point move up to within one error bar from the SLAC data. That would make it fit even better to our curve, but would naturally also be more in accordance with the perturbative QCD prediction of a Q^2 independent ratio.

4. The ratio $R = \sigma_L/\sigma_T$. This ratio vanishes in the quark–parton model according to the Callan–Gross relation. Possible non-zero contributions could come from kinematic mass effects, internal quark transverse momenta and perturbative QCD corrections, but these are supposed to be sizable only at low Q^2 values, which in practice means $x < 0.3$ in, for example, the SLAC data [10].

With a significant diquark content in the nucleon, the situation becomes quite different, since a boson contributes to σ_L. In our model the $(ud)_0$ survives to quite high Q^2 and x values, and should therefore be visible in the data. The SLAC data indeed have σ_L/σ_T significantly different from zero at all x values, and it was noted already in 1979 by Abbott et al. [14] that such an effect can be explained in terms of diquarks. Their model differs from ours though, since the diquarks were thought of as rather large objects ($M^2 = 2$ GeV^2), and not only of spin 0. Such an approach leads to more adjustable parameters than in our model, where the main features have been specified earlier. The stronger suppression by their choice of form factor is, for instance, partly compensated by an ad hoc normalisation to a total of 1.5 diquarks in the nucleon.

We follow the assumption of Abbott et al. that rest masses and parton transverse momenta can be neglected in the Q^2 and x region of interest to us ($Q^2 \geqslant 2$ GeV^2 and $x \gtrsim 0.3$). Then σ_L/σ_T measures directly the ratio of diquarks to quarks (with charges included). On a proton target one obtains:

$$R^p = \tfrac{1}{9}xD(x)F^2(Q^2)$$
$$\times \{\tfrac{4}{9}xu_v(x) + \tfrac{5}{9}xq_D(x)[1 - F^2(Q^2)] + xS(x)\}^{-1}, \tag{7}$$

and correspondingly for a deuterium target, with the $(4/9)xu_v$ above substituted by $(5/18)xu_v$.

The predictions from (7) are plotted in fig. 3 for fixed Q^2 values together with data from SLAC [10] and EMC [26]. It can be seen that a considerable improvement in the measurements of R is needed to al-

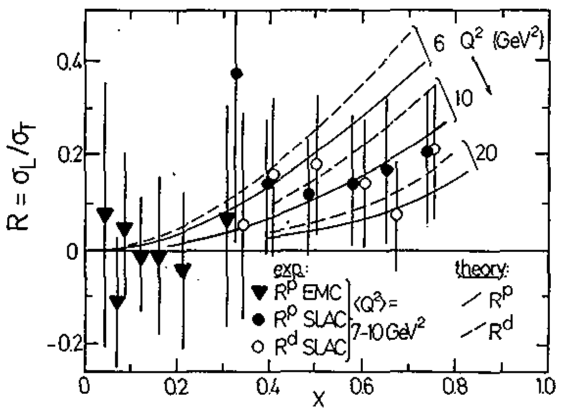

Fig. 3. The ratio $R = \sigma_L/\sigma_T$ of the cross sections for longitudinally to transversely polarised virtual photons versus Bjorken x. The data from SLAC [10] and EMC [26], and show both systematic and statistical errors. Among the SLAC data we have summed only over those Q^2 bins that give $\langle Q^2 \rangle = 7-10$ GeV^2 at each x value, in order to allow comparison with our fixed-Q^2 theoretical results. The curves show the predictions from diquark effects for three fixed Q^2 values and for hydrogen and deuterium targets. Kinematical effects at low x and Q^2 are not included in the theoretical curves.

low a meaningful comparison with our predictions for the Q^2 and x dependences, and with the prediction that R should be different for hydrogen and deuterium targets (and, of course, for heavier nuclear targets). The latter feature would be difficult to understand in a diquark-free model, since both gluon, mass, and k_T effects are flavour independent. The main problem with σ_L/σ_T in perturbative QCD would, however, still be to explain why R is so large to begin with.

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Paper V

Hadron p_T Correlations in Quark Jets

Recently, Aihara *et al.*[1] concluded from their data on 29-GeV e^+e^- annihilation that the creation of diquark-antiquark pairs $(D\bar{D})$ is responsible for $p\bar{p}$ production. Here we would like to object to their additional conclusion that most $D\bar{D}$ pairs are created with other quarks and antiquarks "in between." Such a statement relies on an *ad hoc* assumption in the Lund string model.[2]

There the argument is as follows: (i) Nearby pions in a jet come out mostly back-to-back in the plane perpendicular to the jet. (ii) Therefore, the Lund model sees to it that adjacent hadrons are anticorrelated in the momentum (p_T) transverse to the jet. (iii) Reference 1 shows that a p and its $\bar{p}$ come out mostly on the same side of the jet. (iv) Therefore, a p and a $\bar{p}$ cannot always be nearby in phase space. Often there is a pion produced in between, and the diquarks are created by three two-quark combinations, involving also the quarks of the meson ("popcorn events").

The weak point of this argument is that momentum correlations of a $D\bar{D}$ pair are not the same as those of a $q\bar{q}$ pair. A $q\bar{q}$ pair breaks up with a stronger recoil than a pair of spin-0 diquarks as a result of the lack of spin forces in the $D\bar{D}$ system. This is of particular importance in our own diquark model,[3] where all genuine diquarks have spin 0.

In our view, the $p\bar{p}$ correlation comes about in a simpler way. The p_T of a hadron mirrors the fact that there is a repulsion inside a $q\bar{q}$ or $D\bar{D}$ pair, once its internal color-electric field has been canceled by the opposite external field. This phenomenon manifests itself in two ways in the p_T correlations of hadrons. First, the field by which a pair is created is in some net transverse motion due to the motion of its sources and therefore causes a transverse motion of the whole new $q\bar{q}$ or $D\bar{D}$ pair. Second, the breakup of the new pair adds anticorrelated momentum components to the two new systems.

Depending on the balance between these components, the measured correlation $\alpha = \langle \mathbf{p}_{T;p} \cdot \mathbf{p}_{T;\bar{p}} \rangle / \langle \mathbf{p}_T^2 \rangle$ can be either negative or positive. The parallel component from the field should be about the same for $q\bar{q}$ and $D\bar{D}$, while the antiparallel one is much stronger for $q\bar{q}$ because of the lack of color-magnetic spin forces in the $D\bar{D}$ system. This $q\bar{q}$ repulsion is also indicated by the absence of low-lying mesons with vacuum quantum numbers $J^{PC} = 0^{++}$.

We illustrate this argument with a toy model for the hadron transverse motion. Suppose the field stretched by a $q\bar{q}$ pair in the jet breaks up like a stiff rod when a new $q\bar{q}$ pair is created, and that each piece gets a p_T in proportion to its length. A "rod" with transverse momentum $\mathbf{p}_0$ splits up into two new rods with transverse momenta $\mathbf{p}_1 = x\mathbf{p}_0 + \mathbf{k}$ and $\mathbf{p}_2 = (1-x)\mathbf{p}_0 - \mathbf{k}$, where $\mathbf{k}$ is the repulsive transverse momentum inside the new $q\bar{q}$ pair, and x is the relative coordinate at the point of breakage. Now one gets

$$\alpha = \langle \mathbf{p}_1 \cdot \mathbf{p}_2 \rangle / \langle \mathbf{p}_1^2 \rangle$$
$$= (\tfrac{1}{6}\langle \mathbf{p}_0^2 \rangle - \langle \mathbf{k}^2 \rangle)/(\tfrac{1}{3}\langle \mathbf{p}_0^2 \rangle + \langle \mathbf{k}^2 \rangle).$$

After several repeated $q\bar{q}$ breakups $\langle \mathbf{p}_0^2 \rangle \to 3\langle \mathbf{k}^2 \rangle/2$ and $\alpha_{\pi\pi} \to -\frac{1}{2}$. If a $D\bar{D}$ pair is produced in the last breakup we get $\alpha_{p\bar{p}} > \alpha_{\pi\pi}$ since $\langle \mathbf{k}_D^2 \rangle < \langle \mathbf{k}_q^2 \rangle$. With $\langle \mathbf{k}_D^2 \rangle = 0$ we get $\alpha_{p\bar{p}} = +\frac{1}{2}$. Very similar results are obtained when we take into account that the end point q and $\bar{q}$ have different p_T and that the field carries a transverse-momentum density, which varies smoothly along the "string."

In real life $\alpha_{p\bar{p}}$ and $\alpha_{\pi\pi}$ have smaller magnitudes. First, there might be a weak diquark repulsion due to other than spin forces. Second, an experimental mislabeling of "nearby" hadrons dilutes the true correlations. Third, the two would-be hadrons might have separated earlier than in the last breakup. Finally, the hadrons might have come from resonance decays, which would add an extra p_T component. Such contributions soften the pion and harden the proton p_T spectra. This explains why the mean p_T is higher for protons than for pions.[4]

More detailed predictions would require an event generator, but an obvious test is the one suggested by us earlier,[3] namely to measure the rates of spin-$\frac{3}{2}$ baryons. The popcorn events lead,[2] for instance, to $\Delta/p \simeq 0.6$, while we expect the Δ yield to be suppressed by an order of magnitude.

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[4] This is so because a proton takes nearly all the momentum of a baryon resonance, while a pion takes only half the p_T of a two-pion resonance. If one, for simplicity, assumes that resonance decays contribute, in the mean, a recoil $\langle \mathbf{k}_{\rm res}^2 \rangle \simeq \langle \mathbf{k}_q^2 \rangle$ for both baryons and pions, then one can show that $\alpha_{\pi\pi} \simeq -0.14$ and $\alpha_{p\bar{p}} \simeq +0.17$ in our toy model. Resonances are equally important in the Lund model. See, e.g., H. Aihara *et al.*, Phys. Rev. Lett. **53**, 130 (1984).

Paper VI

ROLE OF DIQUARKS IN THE QUANTUM-CHROMODYNAMICAL PLASMA

Svante Ekelin

December 1986 TRITA-TFY- 86 -16

Abstract

It is suggested that spin-0, color-antitriplet diquarks might occur as a component in the quantum-chromodynamical plasma. Such diquarks would be expected to be favored by Bose statistics at high densities. By treating the plasma as a relativistic ideal gas, it is seen that diquarks indeed carry a large fraction of the total baryon number. An interesting effect is that Bose-Einstein condensation, generally considered to be related chiefly to the low-temperature phenomena of superfluidity and superconductivity, seems to manifest in the QCD plasma at high densities, i.e. high temperatures.

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Currently, much theoretical and experimental effort is being devoted to the study of QCD matter at finite temperature[1]. It is fairly well established that at high temperatures and/or high baryon-number densities, confinement is effectively disabled and chiral symmetry is restored [2]. For systems with vanishing baryon-number density, Monte Carlo simulations of statistical QCD have shown that these transitions occur at the same point [3]. In the case of finite baryon number, the situation is less clear. It has been argued, however, taking instanton effects into account [4] and using finite-temperature QCD sum rules [5] that chiral symmetry restoration cannot precede deconfinement. It is thus possible that there exist three phases of QCD matter : the normal hadronic phase at low temperatures and densities, a plasma phase with deconfined massless quarks, and an intermediate "constituent quark" plasma phase with deconfined massive quarks and massless pions as Goldstone bosons [6]. The chirally symmetric plasma phase, which was universally prevalent during the first microseconds after the big bang and currently is subject to experimental efforts to recreate in relativistic nucleus-nucleus collisions, is generally considered to include light quarks and antiquarks together with gluons as colored components.

However, as has been suggested by several authors [7], there is some evidence that there might exist a bound state of two quarks, a diquark, which could be relatively pointlike at momentum transfers $Q^2 \leq 10\ \text{GeV}^2$. Such spin - 0, color - 3^* objects should, if they exist, also occur as a component in a QCD plasma [8], where they would be expected to be favored by Bose statistics at high densities. Of course, at very high densities, characterised by interquark distances less than $(10\ \text{GeV}^2)^{-1/2}$, diquarks lose their identity and dissolve into quarks. In the intermediate phase, if it exists, diquarks would be expected to be kinematically favored because of constituent mass effects. In the following, for the purpose of illustration of these ideas, we will estimate, using a simple statistical approach, the thermodynamic properties of a plasma including diquarks, and in particular the relative abundance of diquarks.

We will model the chirally symmetric plasma as a relativistic gas of gluons, quarks, antiquarks, diquarks and antidiquarks, and, assuming thermal and chemical equilibrium, use statistical thermodynamics to calculate the properties.

For a statistical ensemble, the grand partition function is

$$\Xi = \mathrm{Tr}\left[\exp\frac{\mu\mathbf{N}-\mathbf{H}}{T}\right], \tag{1}$$

where **H** is the Hamiltonian, **N** the particle-number operator, μ the chemical potential and T the temperature. We always use natural units $\hbar = c = k = 1$.

We will be interested in the high-density regime, and use the free-particle (ideal gas) approximation as suggested by the asymptotic freedom of QCD. The interactions are accounted for [6] simply by introducing a constant energy density B_b and pressure $-B_b$ of the perturbative vacuum, as in the MIT bag model. In this approximation, the trace can easily be performed, in the particle number representation, to give

$$\ln \Xi = \mp \sum_{\mathbf{k}} \ln \left(1 \mp \exp \frac{\mu - E_{\mathbf{k}}}{T} \right), \tag{2}$$

where $E_{\mathbf{k}}$ is the one-particle energy. The upper sign is applicable to bosons and the lower to fermions. Assuming the quantum states $\{\mathbf{k}\}$ to be sufficiently dense [9] , we can approximate the sum by an integral to get (after partial integration)

$$\ln \Xi = \frac{1}{T} \int \sigma(E) \left[\exp \frac{E - \mu}{T} \mp 1 \right]^{-1} \mathrm{d}E. \tag{3}$$

This holds for each component i in the plasma, so the total partition function is

$$\Xi = \prod_i \Xi_i, \tag{4}$$

and the thermodynamic potential is

$$T \ln \Xi = \sum_i T \ln \Xi_i. \tag{5}$$

Thus, we have contributions to the thermodynamic potential from particle type i :

$$T \ln \Xi_i = \int \sigma_i(E) \left[\exp \frac{E - \mu_i}{T} \mp 1 \right]^{-1} \mathrm{d}E, \tag{6}$$

where $\sigma_i(E)$ is the integrated density of one-particle states :

$$\sigma_i(E) = \eta_i \int\limits_{E_i<E} \frac{V\,\mathrm{d}^3p}{(2\pi)^3} = \frac{\eta_i}{6\pi^2} V \left(E^2 - m_i^2\right)^{3/2}, \tag{7}$$

since the one-particle energy $E_i = (p^2 + m_i^2)^{1/2}$, and the state density is

$$\rho_i(E) = \frac{\mathrm{d}}{\mathrm{d}E}\sigma_i(E) = \frac{\eta_i}{2\pi^2} V E \left(E^2 - m_i^2\right)^{1/2}. \tag{8}$$

From

$$\mathrm{d}(T \ln \Xi_i) = S_i\,\mathrm{d}T + P_i\,\mathrm{d}V + N_i\,\mathrm{d}\mu_i, \tag{9}$$

we get

$$P_i = \frac{\eta_i}{6\pi^2} \int\limits_{m_i}^{\infty} \left(E^2 - m_i^2\right)^{3/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E, \tag{10}$$

and, after partial integrations,

$$n_i = \frac{N_i}{V} = \frac{\eta_i}{2\pi^2} \int\limits_{m_i}^{\infty} E \left(E^2 - m_i^2\right)^{1/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E, \tag{11}$$

$$s_i = \frac{S_i}{V} = \frac{\eta_i}{6\pi^2}\frac{1}{T} \int\limits_{m_i}^{\infty} \left(4E^2 - 3E\mu_i - m_i^2\right)\left(E^2 - m_i^2\right)^{1/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E. \tag{12}$$

For the energy density, we get

$$\varepsilon_i = \frac{\eta_i}{2\pi^2} \int\limits_{m_i}^{\infty} E^2 \left(E^2 - m_i^2\right)^{1/2} \left[\exp\frac{E-\mu_i}{T} \mp 1\right]^{-1} \mathrm{d}E. \tag{13}$$

We see that

$$s_i = \frac{1}{T}\left(P_i + \varepsilon_i - \mu_i n_i\right). \tag{14}$$

Now, consider the plasma to contain the components

$$i \in \left\{u, \overline{u}, d, \overline{d}, s, \overline{s}, D, \overline{D}, g\right\}, \tag{15}$$

where D denotes the *(ud)* diquark.

We get the degeneracy factors η_i from the numbers of spin and color states :

$$\eta_f = \eta_{\overline{f}} = 2 \times 3 = 6, \tag{16}$$

$$\eta_D = \eta_{\overline{D}} = 3, \tag{17}$$

$$\eta_g = 2 \times 8 = 16. \tag{18}$$

Our assumption of chemical equilibrium leads to the following relations for the chemical potentials :

$$\mu_D = \mu_u + \mu_d, \tag{19}$$

$$\mu_g = 0, \tag{20}$$

and, for each quark flavor f :

$$\mu_{\overline{f}} = -\mu_f. \tag{21}$$

Assuming the plasma to contain no net strangeness and to be isoscalar, as would be the case in the forward rapidity region in the current CERN and BNL experimental programmes, with ^{16}O, ^{32}S and, possibly, ^{40}Ca beams [10], we get

$$\mu_u = \mu_d \equiv \mu, \tag{22}$$

$$\mu_s = \mu_{\bar{s}} = 0, \tag{23}$$

$$\mu_D = 2\mu. \tag{24}$$

Of course, we have for a plasma with positive baryon number density

$$\mu > 0. \tag{25}$$

Knowing the masses m_i we can now compute the thermodynamic properties and the abundances of the components of the plasma, knowing that the total net baryon number is conserved ($= B$) ; all as functions of T and μ .

We use massless u and d quarks, and set $m_D = m_s = 225$ MeV ; values we got from an analysis of baryon and kaon production data from e^+e^- annihilation [11] . Introducing a small finite mass of the order of 10 MeV to the lightest quarks does not appreciably alter the results.

Gluons are massless and ultrarelativistic, with zero chemical potential, and the integrals can be calculated exactly to give

$$P_g = \frac{1}{3}\varepsilon_g = \frac{8}{45}\pi^2 T^4, \tag{26}$$

$$s_g = \frac{32}{45}\pi^2 T^3, \tag{27}$$

$$n_g = \frac{16}{\pi^2} T^3 \varsigma(3) \approx 1.95\, T^3. \tag{28}$$

For each massless quark flavor $q = u$ or d, the contributions to the pressure, as well as to the baryon-number, energy and entropy densities, can, miraculously, be calculated analytically. We get

$$n_q - n_{\bar{q}} = \mu T^2 + \frac{1}{\pi^2}\mu^3, \tag{29}$$

$$P_q + P_{\bar{q}} = \frac{1}{3}(\varepsilon_q + \varepsilon_{\bar{q}}) = \frac{7}{60}\pi^2 T^4 + \frac{1}{2}\mu^2 T^2 + \frac{1}{4\pi^2}\mu^4, \tag{30}$$

$$s_q + s_{\bar{q}} = \frac{7}{15}\pi^2 T^3 + \mu^2 T. \tag{31}$$

However, as is also the case for the massive plasma components, the separate quark and antiquark contributions cannot be evaluated exactly, so we have calculated all integrals (except for the gluons) numerically and used these analytic results merely as a consistency check.

To get the properties of the plasma solely as functions of the temperature T, we need an additional relation; an evolution equation governing the cooling plasma. We shall assume that cooling through hydrodynamic expansion predominates over cooling through radiation, which we neglect. Thus, we assume that the plasma cools and expands isentropically [12] :

$$\mathrm{d}S = 0. \tag{32}$$

An interesting effect arises due to the inclusion of massive bosons in equilibrium with the quarks in the plasma. As the Bose distribution cannot take on a negative value, we get an upper bound on the boson chemical potential :

$$\mu_D \leq m_D, \tag{33}$$

and, consequently,

$$\mu \leq \frac{m_D}{2}. \tag{34}$$

This does not limit the possible baryon number density, since at $\mu_D = m_D$ a finite fraction of the baryon-number carrying massive bosons can condense in the ground state $p = 0$. These correspond to the first term in (2), and are not counted by the integral in (3). This is an example of the well-known text-book phenomenon of Bose-Einstein condensation [13], traditionally considered to be related to the λ - transition of helium and to Cooper-pair formation [14]. Such condensation is indeed seen to occur at high densities, i.e. high temperatures, in the quantum-chromodynamical plasma. It is interesting to note that this phenomenon, which is assumed to be of relevance to the low-temperature phenomena of superfluidity and superconductivity, also seems to manifest in a QCD plasma at temperatures above 10^{12} K.

At first sight it might seem counter-intuitive that condensation takes place *above* a critical temperature, but this is naturally caused by the increasing density. In the classical idealised case of helium, it is easy to see that as the temperature is lowered at *constant density*, the chemical potential increases and reaches saturation. In this case of *constant entropy*, the chemical potential is seen to increase with temperature.

We have, somewhat arbitrarily, chosen to present results for the case of a constant entropy of 20 units per baryon. This choice might not be too unrealistic [15]. In Fig. 1, the evolution of the plasma in the μ - T plane is shown. It is seen that μ is saturated at its maximum value $m_D/2$ until the temperature has dropped to $T_e = 190$ MeV. After this point, μ drops fairly linearly until chiral symmetry is broken.

Fig. 2 shows the equilibrium abundances of the plasma components. We see that for temperatures above 140 MeV, diquarks are more abundant than quarks. This is due to the fact that at high temperatures, i.e. high densities, quantum-statistical effects start to outweigh the kinematical suppression due to rest mass. In Fig. 3, we display a log-log plot of the total baryon-number density, normalised to that of nuclear matter, versus temperature. It is seen that the behavior is very close to a pure power law. The best fit exponent is 2.98, but $n_B \sim T^3$ also gives an excellent fit, so in this respect we get the same behavior as for a pure massless quark-gluon plasma. The total energy density is shown in Fig. 4 as a function of temperature. For an ideal massless quark-gluon plasma one gets $\varepsilon \sim T^4$, but here we see a somewhat different behavior. The power-law fit is less perfect, and the best-fit exponent is 3.66. This slower rise will, as we shall see, be explained by the increasing fraction of ground-state diquarks.

In Figs. 5a and 5b, we show the fraction of the total baryon number of the plasma carried by the net diquark component. As is seen, this fraction rises with temperature and density. The derivative of this function shows a discontinuity at the temperature $T_e = 190$ MeV, corresponding to a density $(n_B)_e = 6.5\,(n_B)_0$. This is the point beyond which Bose-Einstein condensation in momentum space occurs. The extent to which the diquarks condense is shown in Figs. 6a and 6b, as functions of temperature and density, respectively. We see a sharp initial rise in this fraction, so that only slightly beyond the "critical" limit a significant portion of diquarks are in the same quantum state. Such a "superfluid" plasma component would be likely to remain even in a refined picture where interactions are explicitly taken into account. It might be speculated that collective and coherent phenomena should be drastically enhanced in the plasma in this region. One could, for instance, be more optimistic regarding the possibility of thermalisation. More theoretical work along these lines is called for.

If the condensate should prevail down to the hadronisation regime, one would expect a strong enhancement of the production of multiquark states, such as three-diquark dibaryons, since the diquarks in the condensate all have zero relative velocity, apart from finite-size effects.

It should be noted that we have neglected the component of strange diquarks, *(us)* and *(ds)*, which should be expected to contribute at high temperatures. However, these heavier diquarks, which are perhaps even more pointlike than the *(ud)* diquarks [16], cannot form a condensate, since their equilibrium chemical potential is μ, which cannot exceed $m_D/2$, and $m_{(us)} > m_D$.

All results are given for the case of a constant entropy / baryon of 20 units. One might reasonably ask how sensitive the analysis is to the precise value of this parameter. The phenomenon of Bose-Einstein condensation of the diquark component occurs above a certain temperature, T_e, and above an associated density, $(n_B)_e$. In Figs. 7a and 7b the variation of this condensation temperature and condensation density with the input parameter S/B is shown. It is seen that T_e to a remarkable extent is directly proportional to S/B over the range studied, and that the associated quantity $(n_B)_e$ is very well described by a power-law behavior. The best-fit exponent is 1.96, but $(n_B)_e \sim (S/B)^2$ is also in excellent agreement with the calculated points.

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Footnotes and references

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Figure captions

Fig. 1 The evolution of the plasma in the μ - T plane for a constant entropy per baryon of $S/B = 20$.

Fig. 2 The equilibrium abundances of the plasma components for $S/B = 20$, as functions of temperature. Quarks denote n_u or n_d, similarly for antiquarks.

Fig. 3 The baryon-number density, normalised to that of nuclear matter $(n_B)_0 \approx 0.17\ \mathrm{fm}^{-3}$, as a function of temperature for $S/B = 20$

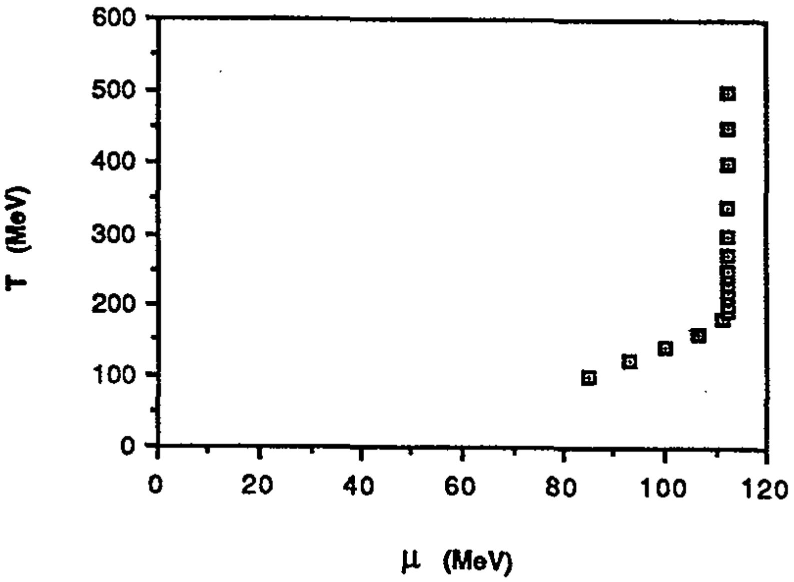
Fig. 4 The total plasma energy density for $S/B = 20$, as a function of temperature.

Fig. 5 The fraction of the total baryon number of the plasma carried by diquarks for $S/B = 20$, versus temperature (5a) and normalised baryon-number density (5b).

Fig. 6 The fraction of diquarks in the plasma that are subject to Bose-Einstein condensation for $S/B = 20$, as functions of temperature (6a) and normalised density (6b).

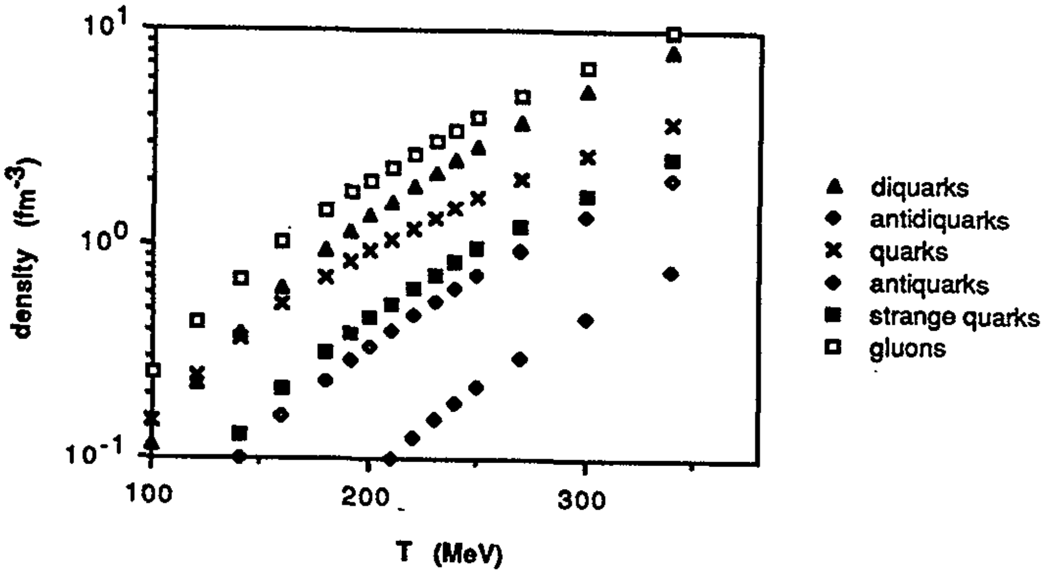
Fig. 7 The variation of the point at which Bose-Einstein condensation of the diquarks in the plasma starts to occur with the input parameter S/B. In Fig. 7a is plotted the condensation temperature T_e, and in Fig. 7b the associated (normalised) condensation density $(n_B)_e/(n_B)_0$, versus S/B.

Fig. 1



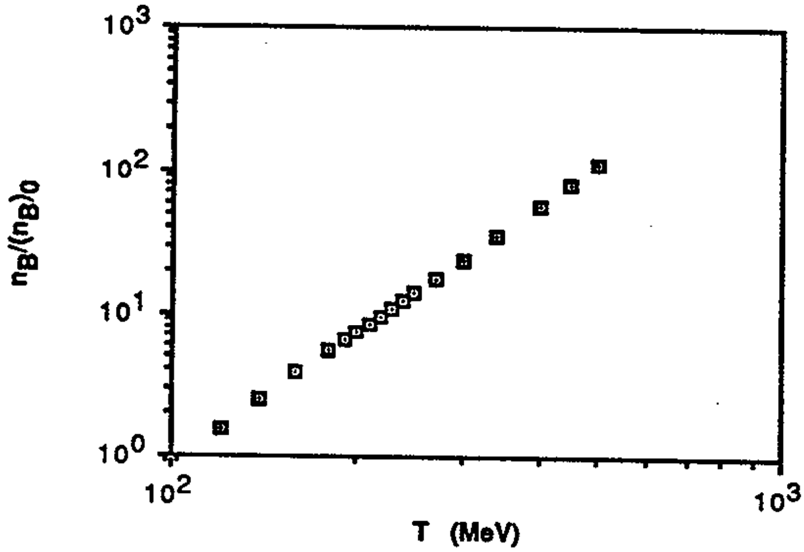
600
500
400
300
200
100
0
T (MeV)
0
20
40
60
80
100
120
μ (MeV)

Fig. 2



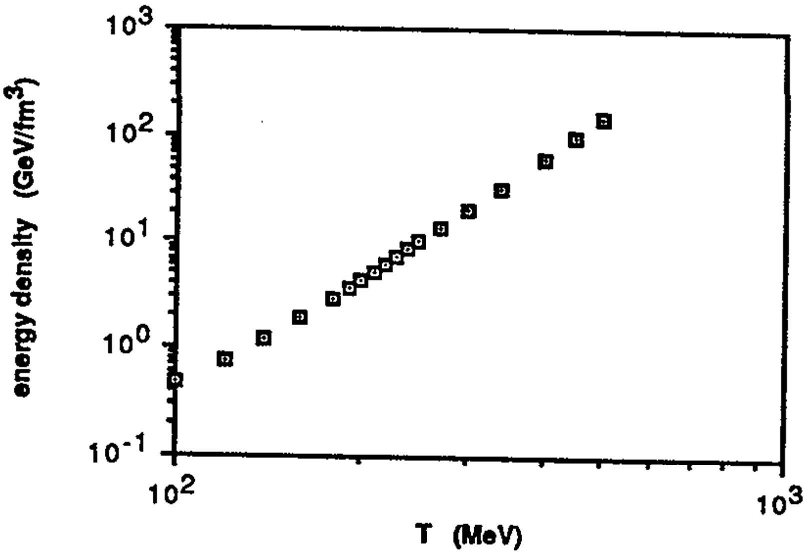
10^1
10^0
10^{-1}
density (fm^{-3})
100
200
300
T (MeV)
diquarks
antidiquarks
quarks
antiquarks
strange quarks
gluons

Fig. 3



10^3
10^2
10^1
10^0
$n_B/(n_B)_0$
10^2
10^3
T (MeV)

Fig. 4



10^3
10^2
10^1
10^0
10^{-1}
energy density (GeV/fm^3)
10^2
10^3
T (MeV)

Fig. 5a

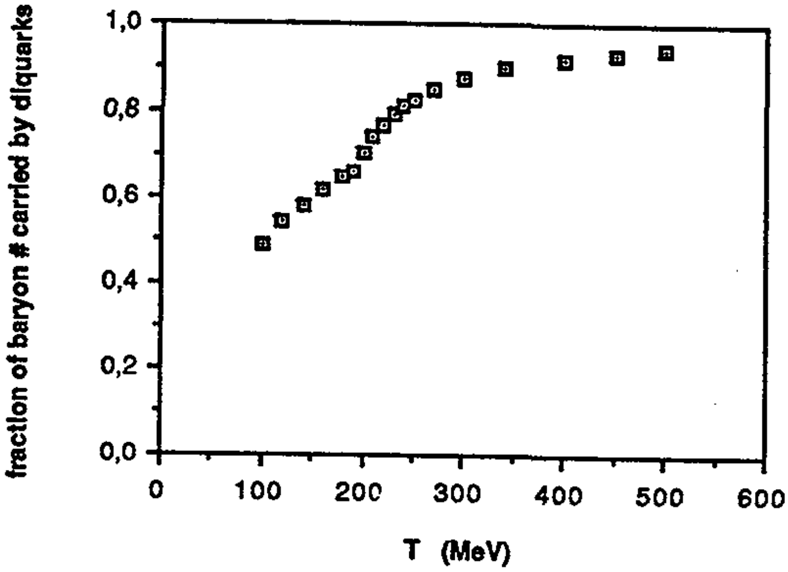

fraction of baryon # carried by diquarks
1,0
0,8
0,6
0,4
0,2
0,0
0
100
200
300
400
500
600
T (MeV)

Fig. 5b

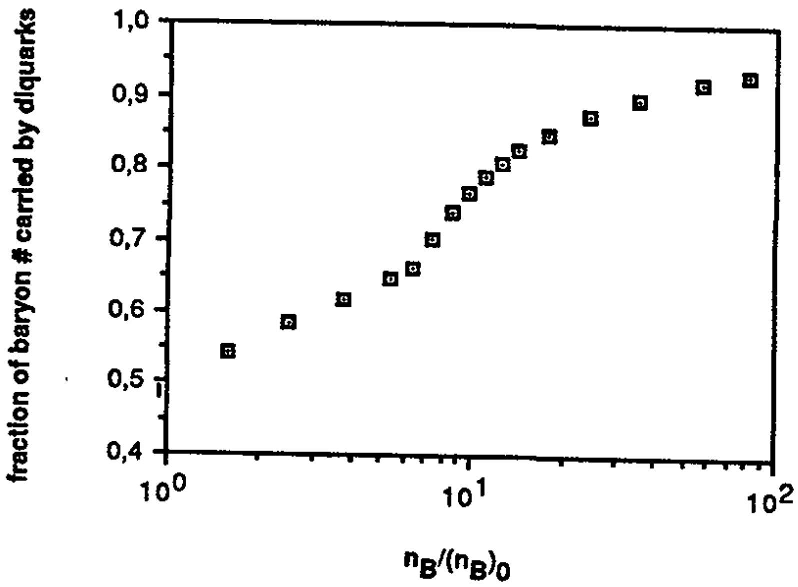

fraction of baryon # carried by diquarks
1,0
0,9
0,8
0,7
0,6
0,5
0,4
10^0
10^1
10^2
$n_B/(n_B)_0$

Fig. 6a

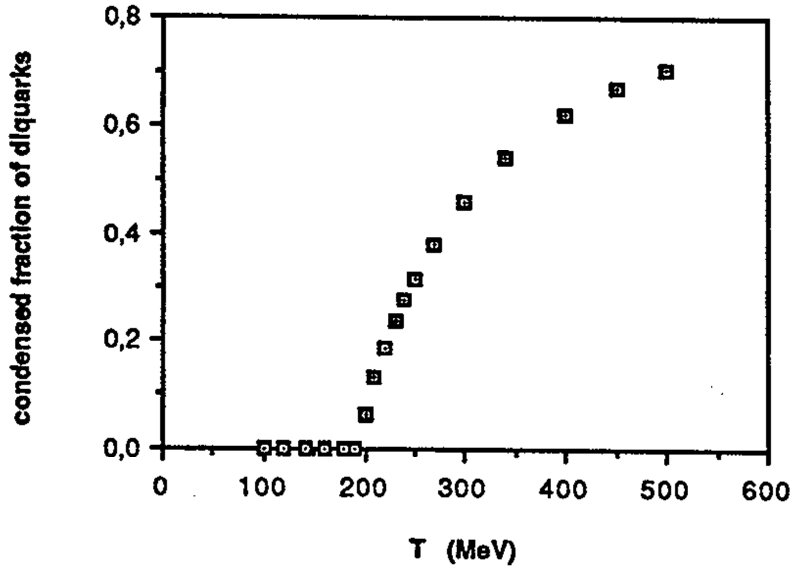

0,8
0,6
0,4
0,2
0,0
condensed fraction of diquarks
0
100
200
300
400
500
600
T (MeV)

Fig. 6b

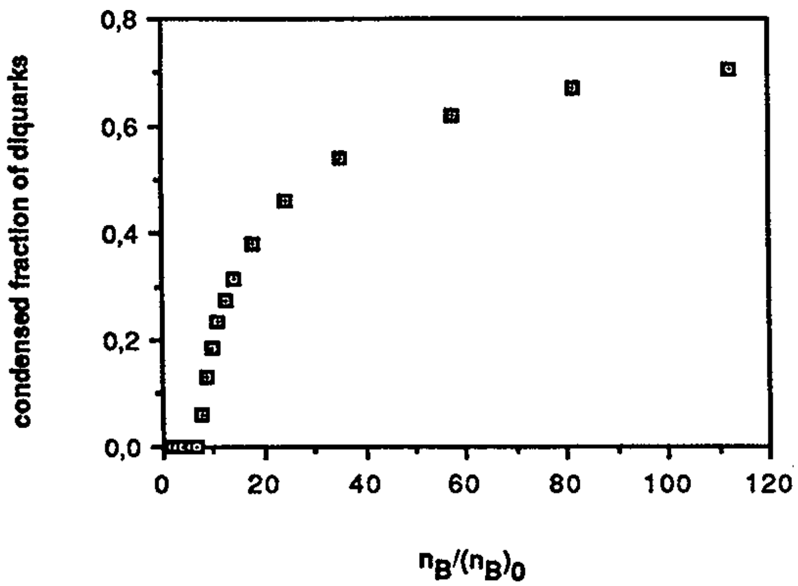
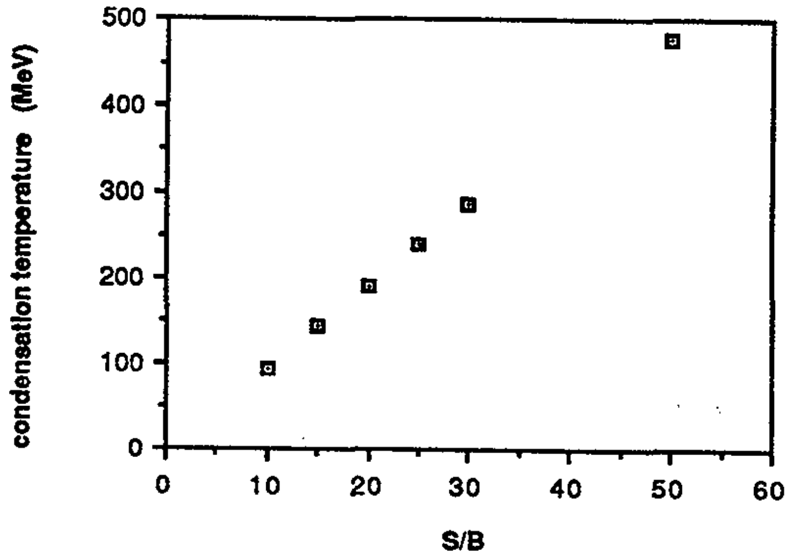
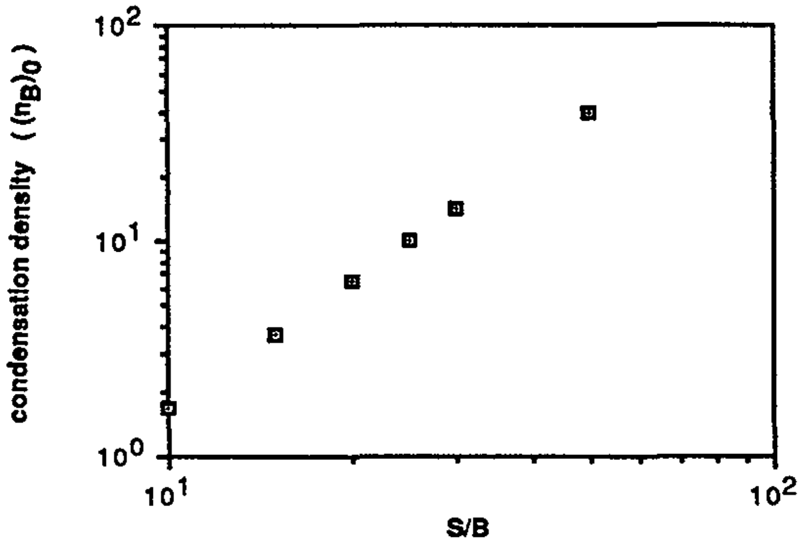

0,8
0,6
0,4
0,2
0,0
condensed fraction of diquarks
0
20
40
60
80
100
120
nB/(nB)0

Fig. 7a



condensation temperature (MeV)
500
400
300
200
100
0
0
10
20
30
40
50
60
S/B

Fig. 7b



condensation density ($(n_B)_0$)
10^2
10^1
10^0
10^1
10^2
S/B

Each color connects
to create the boat
which rocks the race.

(James Douglas Morrison)